Erratum, "Canonical vector heights on K3 surfaces with Picard number three – an argument for nonexistence," *Math. Comp.*, **73**(248), 2019 – 2025 (2004).

(Please see the update below.)

As is pointed out by the reviewer in *Mathematical Reviews*, there is a "gap" in the proof that the example of the paper has Picard number three. The reviewer's description is generous. Thank you.

When I wrote the paper, I was of the mistaken belief that there was only one way in which the Picard number could be increased (the way described in the paper). It was not until after its publication that I became aware that there are in fact a countably infinite number of ways in which the Picard number could increase, and that finding (significant) upper bounds on the Picard number for a particular example is a non-trivial problem. Since the generic surface in this class has Picard number three, one could argue that it probably has Picard number three, but that argument is not too convincing. For example, contrast it with the argument that "since generic numbers are transcendental, if a person is asked to pick a number at random, they will probably choose a transcendental number."

After being made aware of the gap, I briefly hoped that I could work around it, but it soon became evident that the argument depends in a crucial way on having a complete basis.

I will therefore try to prove that the Picard number for the example is in fact three, or barring that, find a surface of this type whose Picard number is three and run the experiment again.

For the moment, it would seem that the argument for nonexistent is less convincing than originally anticipated. One might even still hope that canonical vector heights exist.

April 22, 2005

Update:

Ronald van Luijk has a method for calculating upper bounds on the Picard number. He was kind enough to check the surface of this paper, but was only able to get an upper bound of six, so we now know $3 \leq n \leq 6$.

Rather than try to sharpen this bound, we instead found a different surface for which we (or rather, Ronald) could prove $n = 3$. We then ran the same experiment on this new surface. The conclusions were unchanged: Except in very special cases, it is unlikely that a K3 surface with Picard number at least 3 will admit a canonical vector height.

The special cases I think might be those where the group of automorphisms for the surface is, up to finite groups, isomorphic to $\mathbb{Z}^m$ for some m. I do not know whether it is possible for this to happen with $m \geq 2$.

These results will soon appear in a preprint on this site.

February 7, 2006