

### Sample Exam 3, MAT 283

See our website for solutions.

1. Expect a question like one of the following:

(a) Evaluate

$$\int \int_R (x + 2y) dA$$

where  $R$  is the region bounded by  $y = 2x^2$  and  $y = 1 + x^2$ .

(b) Evaluate

$$\int_0^1 \int_x^1 \sin(y^2) dy dx$$

by switching the order of integration.

(c) Evaluate

$$\int \int \int_R \sqrt{x^2 + z^2} dV$$

where  $R$  is the region bounded by  $y = x^2 + z^2$  and  $y = 4$ .

2. Expect a question like one of the following:

(a) Use a double integral to find the area enclosed by one loop of the four leaved rose  $r = \cos 2$ .

(b) Find the area of the part of the paraboloid  $z = x^2 + y^2$  that lies under the plane  $z = 9$ .

3. Expect a question like one of the following:

(a) Evaluate

$$\int \int \int_B e^{(x^2+y^2+z^2)^{3/2}} dV$$

where  $B$  is the unit ball

$$B = \{(x, y, z) : x^2 + y^2 + z^2 \leq 1\}.$$

(b) Evaluate

$$\int \int_R e^{\left(\frac{x+y}{x-y}\right)} dA$$

where  $R$  is the trapezoidal region with vertices  $(1, 0)$ ,  $(2, 0)$ ,  $(0, -2)$ , and  $(0, -1)$ , by making the change of variables

$$x = \frac{1}{2}(u + v) \quad \text{and} \quad y = \frac{1}{2}(u - v).$$

4. Expect a question like one of the following:

(a) Evaluate

$$\int_C (2 + x^2y) ds$$

where  $C$  is the upper half of the unit circle  $x^2 + y^2 = 1$ .

(b) Evaluate

$$\int_C y^2 dx + x dy$$

where  $C$  is the arc of the parabola  $x = 4 - y^2$  from  $(-5, -3)$  to  $(0, 2)$ .

5. Let  $\vec{F}(x, y) = (3 + 2xy, x^2 - 3y^2)$ . Show that  $\vec{F}$  is conservative and find a potential function  $P$  such that  $\vec{\nabla}P = \vec{F}$ . Evaluate

$$\int_C \vec{F} \cdot d\vec{x}$$

for  $C$  the curve  $\vec{x}(t) = (e^t \sin t, e^t \cos t)$ ,  $t \in [0, \pi]$ .