

Sample Exam 1, MAT 253

See our website for partial solutions.

1. (20 points) Let

$$S = \{(x, y) : x, y \in \mathbb{R}\}$$

and define addition and scalar multiplication on S as follows:

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, 0)$$

$$c(a_1, a_2) = (ca_1, 0).$$

Is this a vector space? Justify your answer.

2. (30 points) Let

$$f_1(x) = x^2 + 3x - 2$$

$$f_2(x) = 2x^2 + 5x - 3$$

$$f_3(x) = -x^2 - 4x + 4.$$

Find scalars d_1 , d_2 and d_3 in terms of a , b and c such that

$$d_1f_1 + d_2f_2 + d_3f_3 = ax^2 + bx + c.$$

3. (10 point) Let f_1 , f_2 , and f_3 be as in Question 2. Is $S = \{f_1, f_2, f_3\}$ a basis of $P_2(\mathbb{R})$? Explain.

4. (20 points) Let

$$W = \{(a_1, a_2, a_3, a_4, a_5) \in \mathbb{R}^5 : a_1 + a_3 + a_5 = 0, a_2 = a_4\}$$

Show that W is a subspace of \mathbb{R}^5 and find a basis of W . What is the dimension of W ?

5. (10 points) Let V be a vector space over \mathbb{R} . Let $a, b \in \mathbb{R}$ and $\vec{u}, \vec{v} \in V$. Prove that

$$(a + b)(\vec{u} + \vec{v}) = a\vec{u} + a\vec{v} + b\vec{u} + b\vec{v}.$$