MAT 181 Calculus I **Quiz #2 Solution**
Prof. H. Cho (Sec. 2.4-2.6)

1. Differentiate the function.

(a) $f(x)=\sqrt{x}(x-1)=x^{3/2}-x^{1/2}$. So, $f'(x)=\frac{3}{2}x^{1/2}-\frac{1}{2}x^{-1/2}=\underline{\frac{1}{2}x^{-1/2}(3x-1)}$ or $\underline{\frac{3x-1}{2\sqrt{x}}}$.

(b) $y=\frac{\sin\theta}{2}+\frac{c}{\theta}$, c is a constant.

Since $y=\frac{1}{2}\sin\theta+c\theta^{-1}$, $y'=\frac{1}{2}\cos\theta+c(-1)\theta^{-2}=\underline{\frac{\cos\theta}{2}-\frac{c}{\theta^2}}$ or $\underline{\frac{\theta^2\cos\theta-2c}{2\theta^2}}$.

2. Find an equation of the tangent line to the curve at the given point as follows.

$y=\frac{1}{(1+x^2)}$ at P = (-1, ½). This curve is called a *witch of Maria Agnesi*.

Sol: $f'(x)=\frac{(1+x^2)(0)-1(2x)}{(1+x^2)^2}=\frac{-2x}{(1+x^2)}$. So the slope of the tangent line at the point (-1, ½)

is $f'(-1)=\frac{2}{2^2}=\frac{1}{2}$ and its equation is $\underline{y-\frac{1}{2}=\frac{1}{2}(x+1)}$ or $\underline{y=\frac{1}{2}x+1}$.

3. Find the derivatives of the function.

(a) *f'* for $f(x)=\sqrt{\sin x}$. Let $u=g(x)=\sin x$ and $y=f(u)=\sqrt{u}$.

Then $\frac{dy}{dx}=\frac{dy}{du}\bullet\frac{du}{dx}=\frac{1}{2}u^{-1/2}\cos x=\frac{\cos x}{2\sqrt{u}}=\underline{\frac{\cos x}{2\sqrt{\sin x}}}$.

(b) *y''* for $y=(1-7t)^6$. Since $y'=6(1-7t)^5(-7)=-42(1-7t)^5$,

$y''=-42(5)(1-7t)^4(-7)=\underline{1470(1-7t)^4}$

4. Find *dy/dx* by implicit differentiation.

(a) $y^5+x^2y^3=1+x^4y$

$$\frac{d}{dx}\left(y^5+x^2y^3\right)=\frac{d}{dx}\left(1+x^4y\right)$$

$$\Leftrightarrow 5y^4y'+x^2 3y^2y'+2xy^3=0+x^4y'+y4x^3$$

$$\Leftrightarrow y'(5y^4+3x^2y^2-x^4)=4x^3y-2xy^3$$

$$\Leftrightarrow y'=\frac{4x^3y-2xy^3}{5y^4+3x^2y^2-x^4}$$

(b) $4\cos x\sin y=1$

$$\frac{d}{dx}\left(4\cos x\sin y\right)=\frac{d}{dx}(1)$$

$$\Leftrightarrow 4\left[\cos x\cdot\cos y\cdot y'+\sin y(-\sin x)\right]=0$$

$$\Leftrightarrow y'(4\cos x\cdot\cos y)=4\sin x\sin y$$

$$\Leftrightarrow y'=\frac{4\sin x\sin y}{4\cos x\cdot\cos y}=\tan x\tan y$$