Limits and Continuity for functions of several Variables

1. Find the limits and say whether the function is continuous at the point in question:

(a) $\lim\limits_{(x,y) \to (2,1)} (x + 3y^2)$

(b) $\lim\limits_{(x,y) \to (2,4)} \dfrac{x + y}{x - y}$

(c) $\lim\limits_{(x,y) \to (0,1)} \dfrac{\arcsin(x / y)}{1 + xy}$

(d) $\lim\limits_{(x,y,z) \to (1,2,5)} \sqrt{x + y + z}$

2. Say whether the following limits exist or not –prove what you say is correct:

(a) $\lim\limits_{(x,y) \to (0,0)} \dfrac{y}{x^2 + y^2}$

(b) $\lim\limits_{(x,y) \to (0,0)} \dfrac{-xy^2}{x^2 + y^4}$

(c) $\lim\limits_{(x,y) \to (0,0)} \dfrac{xy}{x^2 + y^2}$

3.

Find each limit by using polar coordinates if the limit exists. Do this by substituting x $=$ r$\cos\theta$, y $=$ r$\sin\theta$, and note that (x, y) $\to$ (0,0) is equivalent to saying r $\to$ 0:

(a) $\lim\limits_{(x,y) \to (0,0)} \dfrac{\sin(x^2 + y^2)}{x^2 + y^2}$

(b) $\lim\limits_{(x,y) \to (0,0)} \dfrac{x^3 + y^3}{x^2 + y^2}$

Solutions for Limits/Continuity - functions of Several Variables.

① (a) $\lim_{(x,y)\to(2,1)} (x+3y^2) = 2+3(1)^2 = 5$

(b) $\lim_{(x,y)\to(2,4)} \frac{x+y}{x-y} = \frac{2+4}{2-4} = \frac{6}{-2} = -3$

(c) $\lim_{(x,y)\to(0,1)} \frac{\arcsin(x/y)}{1+xy} = \frac{\text{Arcsin}(0/1)}{1+0\cdot 1} = \frac{0}{1} = 0$

(d) $\lim_{(x,y,z)\to(1,2,5)} \sqrt{x+y+z} = \sqrt{1+2+5} = \sqrt{8} = 2\sqrt{2}$

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② (a) $\lim_{(x,y)\to(0,0)} \frac{y}{x^2+y^2}$ DNE.

Approaching (0,0) along x-Axis ($y=0$) gives:

$\lim_{(x,y)\to(0,0)} \frac{y}{x^2+y^2} = \lim_{x\to 0} \frac{0}{x^2+0} = 0$

Approaching (0,0) along y-Axis ($x=0$) gives:

$\lim_{(x,y)\to(0,0)} \frac{y}{x^2+y^2} = \lim_{y\to 0} \frac{y}{0^2+y^2} = \lim_{y\to 0} \frac{1}{y}$ DNE.

(b) $\lim_{(x,y)\to(0,0)} \frac{-xy^2}{x^2+y^4}$ DNE. Along line $x=0$: $\lim_{y\to 0} \frac{-(0)y^2}{0^2+y^4} = 0$

Along line $y=\sqrt{x}$: $\lim_{x\to 0} \frac{-x(\sqrt{x})^2}{x^2+(\sqrt{x})^4} = -\lim_{x\to 0} \frac{x^2}{x^2+x^2} =$

$-\lim_{x\to 0} \frac{x^2}{2x^2} = -\lim_{x\to 0} \frac{1}{2} = -\frac{1}{2}$

(c) $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ DNE ;

Along $x=0$: $\lim_{y\to 0} \frac{0(y)}{0^2+y^2} = 0$

Along $y=x$: $\lim_{x\to 0} \frac{x^2}{x^2+x^2} = 1/2$

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(3) (a) $\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} = \lim_{r\to 0} \frac{\sin(r^2\cos^2\theta + r^2\sin^2\theta)}{r^2\cos^2\theta + r^2\sin^2\theta}$

$= \lim_{r\to 0} \frac{\sin(r^2)}{r^2} = 1$.

(b) $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2} = \lim_{r\to 0} \frac{r^3(\cos^3\theta + \sin^3\theta)}{r^2}$

$= \lim_{r\to 0} r(\cos^3\theta + \sin^3\theta) = 0$ for every value of θ.