Particular Partial Derivatives

1. Find both first partial derivatives of each function:

   (*a*) $z = 2x - 3y + 5$

   (*b*) $z = x\sqrt{y}$

   (*c*) $z = x^2 e^{2y}$

   (*d*) $z = \ln(x^2 + y^2)$

   (*e*) $z = \ln\dfrac{x+y}{x-y}$

   (*f*) $z = e^{-(x^2 + y^2)}$

   (*g*) $z = \sqrt{x^2 + y^2}$

   (*h*) $z = \tan(2x - y)$

   (*i*) $z = e^y \sin xy$

   (*j*) $g(x,y) = \int_x^y (t^2 - 1)\,dt$

2. Find the slopes in the x and y-directions at the indicated point:

   (*a*) $g(x,y) = 4 - x^2 - y^2$, *at* $(1,1,2)$

   (*b*) $h(x,y) = x^2 - y^2$ *at* $(-2,1,3)$

   (*c*) $z = e^{-x} \cos y$ *at* $(0,0,1)$

3. The temperature at any point (x,y) on a steel plate is given by the function

   $T(x,y) = 500 - .6x^2 - 1.5y^2$

   where x and y are in meters. At the point (2, 3), find the rate of change of the temperature with respect to the distance moved along the plate in the directions of the x and y - axes.

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Solutions for Particular Partial Derivatives

1. (a) $z = 2x - 3y + 5 \Longrightarrow z_x = 2, \; z_y = -3$

(b) $z = x\sqrt{y} \Longrightarrow z_x = \sqrt{y}, \; z_y = x \cdot \frac{1}{2\sqrt{y}} = \frac{x}{2\sqrt{y}}$

(c) $z = x^2 e^{2y} \Longrightarrow z_x = 2x e^{2y}, \; z_y = 2x^2 e^{2y}$

(d) $z = \ln(x^2+y^2) \Longrightarrow z_x = \frac{1}{x^2+y^2} \cdot 2x = \frac{2x}{x^2+y^2}$ and

$$z_y = \frac{2y}{x^2+y^2}$$

(e) $z = \ln\left(\frac{x+y}{x-y}\right) = \ln(x+y) - \ln(x-y) \Longrightarrow z_x = \frac{1}{x+y} - \frac{1}{x-y}$

and $z_y = \frac{1}{x+y} - \frac{1}{x-y}(-1) = \frac{1}{x+y} + \frac{1}{x-y}$

(f) $z = e^{-(x^2+y^2)} \Longrightarrow z_x = -2x e^{-(x^2+y^2)}, \; z_y = -2y e^{-(x^2+y^2)}$

(g) $z = \sqrt{x^2+y^2} \Longrightarrow z_x = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$ and

$$z_y = \frac{y}{\sqrt{x^2+y^2}}$$

(over →)

(h) $z = \tan(2x-y) \Longrightarrow z_x = 2\sec^2(2x-y)$ and

$$z_y = -\sec^2(2x-y)$$

(i) $z = e^y \sin xy \Longrightarrow z_x = y e^y \cos xy$ and

$$z_y = e^y[(\cos xy)x] + e^y \sin xy =$$
$$x e^y \cos xy + e^y \sin xy$$

(j) $g(x,y) = \int_x^y (t^2-1)\,dt$

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challenge: Can you make a conjecture about how to find $\frac{\partial}{\partial x} \int_{u(x)}^{v(y)} h(t)\,dt$ and $\frac{\partial}{\partial y} \int_{u(x)}^{v(y)} h(t)\,dt$ in general even when you do not know how, explicitly, to calculate $\int h(t)\,dt$? Can you verify your conjecture gives the correct answer in a variety of different examples?

Can you extend your conjecture to $\frac{\partial}{\partial x} \int_{u(x,y)}^{v(x,y)} h(t)\,dt$ and $\frac{\partial}{\partial y} \int^{v(x,y)} h(t)\,dt$?

slope in x-direction

2 (a) $g(x,y) = 4 - x^2 - y^2$, so $g_x\big|_{(1,1,2)} = -2x\big|_{(1,1,2)} = -2(1) = -2$

$g_y\big|_{(1,1,2)} = -2y\big|_{(1,1,2)} = -2(1) = -2$

(b) $h(x,y) = x^2 - y^2$, so $h_x\big|_{(-2,1,3)} = 2x\big|_{(-2,1,3)} = 2(-2) = -4$

$h_y\big|_{(-2,1,3)} = -2y\big|_{(-2,1,3)} = -2(1) = -2$

(c) $z = e^{-x}\cos y$, so $z_x\big|_{(0,0,1)} = -\cos y\, e^{-x}\big|_{(0,0,1)} = -\cos(0)\cdot e^{-0}$

$= -1$

$z_y\big|_{(0,0,1)} = e^{-x}(-\sin y)\big|_{(0,0,1)} = 0$

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3 $T(x,y) = 500 - .6x^2 - 1.5y^2$

rate of Temp change in x-direction $= T_x\big|_{(2,3)} = -1.2x\big|_{(2,3)} =$

$-2.4°$ per meter.

rate of temp. change in y-direction $= T_y\big|_{(2,3)} = -3y\big|_{(2,3)} =$

$-9°$/meter.