Differing Differentials

1. Find the total differential:

   $(a)\ z = 3x^2y^3$

   $(b)\ z = \dfrac{-1}{x^2 + y^2}$

   $(c)\ z = x\cos y - y\cos x$

2. For each of the following functions, evaluate f(1, 2) and f(1.05, 2.1) and calculate ΔZ. Then use the total differential dZ to approximate ΔZZ:

   $(a)\ f(x,y) = 9 - x^2 - y^2$

   $(b)\ f(x,y) = x\sin y$

3. The radius r and height h of a right circular cylinder are measured with possible errors of 4% and 2%. Approximate the maximum possible percent error in measuring the volume.

4. A triangle is measured and two adjacent sides are found to measure 3 and 4 inches long, with an included angle of $\pi/4$ radians. The possible errors in measurements are 1/16 inch in the sides and .02 radian in the angle. Approximate the maximum possible error in the computation of the area.

5. Electrical power is given by the equation:

   $$P = E^2 / R$$

   Where E is voltage and R is resistance. Approximate the maximum percent error in calculating power if 200 volts is applied to a 400 ohm resister and the possible percent errors in measuring E and R are 2% and 3%.

Solutions for Differing Differentials

(1) (a) $z = 3x^2y^3$, $z_x = 6xy^3$, $z_y = 9x^2y^2$, so

$$dz = z_x\,dx + z_y\,dy = \underline{6xy^3\,dx + 9x^2y^2\,dy}$$

(b) $z = \frac{-1}{x^2+y^2} = -(x^2+y^2)^{-1} \Longrightarrow$

$z_x = -(-1)(x^2+y^2)^{-2} \cdot 2x = \frac{2x}{(x^2+y^2)^2}$ and

$z_y = \frac{2y}{(x^2+y^2)^2}$, so $dz = \frac{2x}{(x^2+y^2)^2}dx + \frac{2y}{(x^2+y^2)^2}dy$

(c) $z = x\cos y - y\cos x$, $z_x = \cos y + y\sin x$,

$z_y = -x\sin y - \cos x$, $dz = (\cos y + y\sin x)\,dx + (-x\sin y - \cos x)\,dy$

(2) (a) $f(x,y) = 9 - x^2 - y^2$. $f(1,2) = 4$, $f(1.05, 2.1) = 3.4875$

So $\Delta z = 3.4875 - 4 = -.5125$

$dz = -2x\,dx + -2y\,dy =$

$-2(1)(.05) + -2(2)(.1) = -.5$

(over →)

(b) $f(x,y) = x\sin y$, $f(1,2) = \sin 2$, $f(1.05, 2.1) = 1.05\sin 2.1$

so $\Delta z \doteq -.00293$

$dz = \sin y\,dx + x\cos y\,dy = (\sin 2)(.05) + (\cos 2)(.1)$

$\doteq .00385$

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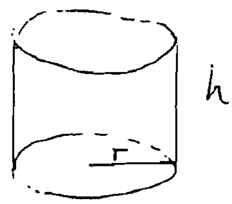
(3) $V = \pi r^2 h$



$$dV = \frac{\partial V}{\partial r}dr + \frac{\partial V}{\partial h}dh =$$

$$2\pi r h\,dr + \pi r^2 dh$$

$$\text{so } \frac{dV}{V} = \frac{2\pi r h\,dr + \pi r^2 dh}{\pi r^2 h} = \frac{2}{r}dr + \frac{dh}{h}$$

$$= 2\left(\frac{dr}{r}\right) + \frac{dh}{h} = 2(\pm.04) + (\pm.02)$$

$$= 2(\pm.02\cdot 2) + (\pm.02) = (\pm.02)[4+1]$$

$$= \pm.02(5) = \pm.1$$

i.e. max % error $= \frac{dV}{V}\cdot 100\% = \pm 10\%$ relative error.

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(4)

Area $= A = \frac{1}{2} ab \sin\theta$

$$dA = \frac{\partial A}{\partial a} da + \frac{\partial A}{\partial b} db + \frac{\partial A}{\partial \theta} d\theta$$

$$= \frac{1}{2}\left[b\sin\theta \, da + a\sin\theta \, db + ab\cos\theta \, d\theta\right]$$

$$= \frac{1}{2}\left[4(\sin\tfrac{\pi}{4})(\pm\tfrac{1}{16}) + 3(\sin\tfrac{\pi}{4})(\pm\tfrac{1}{16}) + 12(\cos\tfrac{\pi}{4})(\pm .02)\right]$$

$$= \pm\frac{1}{2}\left[\frac{1}{4\sqrt{2}} + \frac{3}{16\sqrt{2}} + \frac{.24}{\sqrt{2}}\right] \doteq \pm .24 \text{ in}^2.$$

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(5) $P = \dfrac{E^2}{R}$, where E is in Volts and R is in Ohms.

$$dp = \frac{2E}{R} dE + \frac{-E^2}{R^2} dR \Longrightarrow$$

$$\frac{dP}{P} = \frac{2E/R}{E^2/R} dE + \frac{-E^2/R^2}{E^2/R} dR =$$

$$\frac{2}{E} dE - \frac{1}{R} dR$$

|Max relative Error| = Max $|2(\pm .02) - (\pm .03)| = .07$

So Max percentage error = ~~.069~~ $\pm$ 7 %