Double Integrals and Volume

① Sketch the region R and evaluate $\iint_R f(x,y)\,dA$:

(a) $\int_0^2 \int_0^1 (1+2x+2y)\,dy\,dx$

(b) $\int_0^6 \int_{y/2}^3 (x+y)\,dx\,dy$

(c) $\int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} (x+y)\,dy\,dx$

② Evaluate each integral $\iint_R f(x,y)\,dA$:

(a) $\iint_R xy\,dA$. R is the rectangle with vertices at $(0,0), (0,5), (3,5), (3,0)$.

(b) $\iint_R \frac{y}{x^2+y^2}\,dA$, R is the triangle bounded by $y=x,\ y=2x,\ x=2$.

(over →)

(c) $\iint_R x\,dA$, where R is the Sector of a circl

in the first quadrant bounded by $y = \sqrt{25-x2}$,

$3x - 4y = 0$, $y = 0$.

③ Sketch the Solid described and use a double integral to find the volume bounded by the solid:

(a) The Solid bounded by $0 \le x \le 4$, $0 \le y \le 2$ and the plane $z = y/2$.

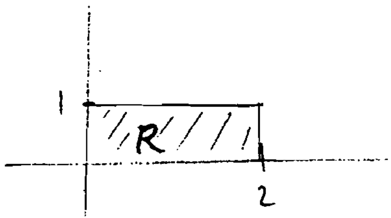
(b) The solid bounded by $y = x$, $y = 2$, and the plane $z = 6 - x - y$, and $x = 0$

(c) The Solid bounded by the plane $2x + 3y + 4z = 12$ and the coordinate planes in the first octant.

(d) The Solid bounded by $y = x$, $y = 1$, and the Surface $z = 1 - xy$

Solutions for Double integrals and Volume

1 (a)

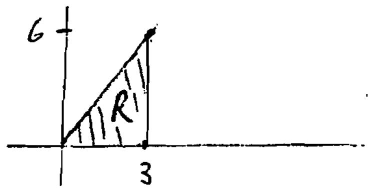


$$\int_0^2 \int_0^1 (1+2x+2y)\,dy\,dx =$$

$$\int_0^2 \left[y+2xy+y^2\right]_{y=0}^{1} dx = \int_0^2 (1+2x+1)\,dx = \int_0^2 (2+2x)\,dx$$

$$= 2x + x^2 \Big]_0^2 = 8$$

(b)



$$\int_{y=0}^{6} \left[\int_{x=y/2}^{3} (x+y)\,dx\right] dy$$

$$= \int_{y=0}^{6} \left[\frac{x^2}{2}+xy\right]_{x=y/2}^{3} dy = \int_{y=0}^{6} \left[\left(\frac{9}{2}+3y\right)-\left(\frac{y^2}{8}+\frac{y^2}{2}\right)\right] dy$$

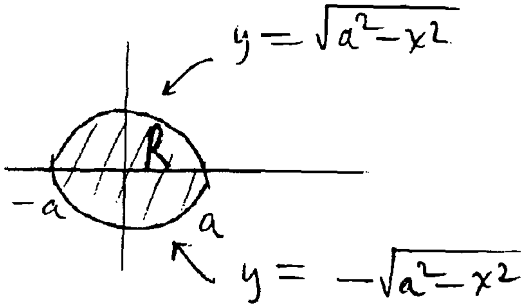
$$= \int_{y=0}^{6} \left(\frac{9}{2}+3y-\frac{5}{8}y^2\right) dy = \frac{9}{2}y + \frac{3y^2}{2} - \frac{5}{24}y^3 \Big]_{y=0}^{6}$$

$$= \frac{9}{2}(6) + \frac{3(36)}{2} - \frac{5}{24}(6)^3 =$$

$$27 + 54 - 45 = 36$$

(OVER→)

(c)

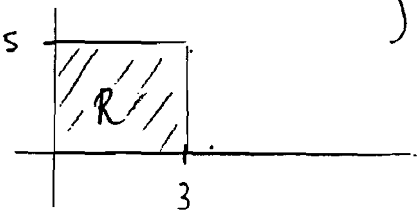


$$\int_{-a}^{a}\left[\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}(x+y)\,dy\right]dx=\int_{-a}^{a}\left[xy+\frac{y^2}{2}\right]_{y=-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}dx$$

$$=\int_{-a}^{a}\left\{\left(x\sqrt{a^2-x^2}+\frac{a^2-x^2}{2}\right)-\left(-x\sqrt{a^2-x^2}+\frac{a^2-x^2}{2}\right)\right\}dx$$

$$=\int_{-a}^{a}2x\sqrt{a^2-x^2}\,dx=0$$

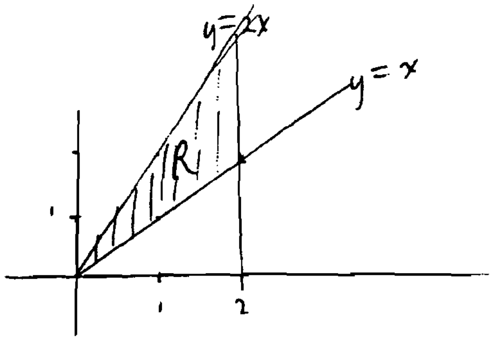
(2) (a)



$$\iint_R xy\,dA = \int_0^3\int_0^5 xy\,dy\,dx$$

$$= \boxed{225/4}$$

(b)



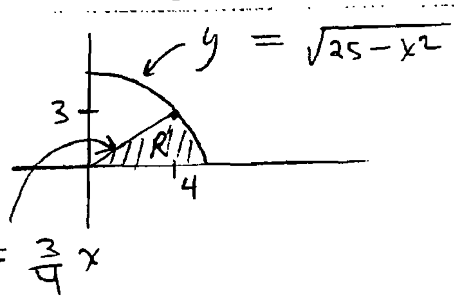
$$\iint_R \frac{y}{x^2+y^2}\,dy\,dx$$

$$= \int_{x=0}^{2}\left[\int_{y=x}^{2x} \frac{y}{x^2+y^2}\,dy\right]dx = \int_{t=0}^{2}\left[\tfrac{1}{2}\ln(x^2+y^2)\right]_{y=x}^{2x} dx$$

$$= \frac{1}{2}\int_0^2 \left[\ln(x^2+4x^2) - \ln(x^2+x^2)\right] dx = \frac{1}{2}\int_0^2 (\ln 5x^2 - \ln 2x^2)\,dx$$

$$= \frac{1}{2}\int_0^2 \left(\ln \frac{5}{2}\right) dx = \frac{1}{2}\ln\frac{5}{2}\int_0^2 dx = \ln 5/2$$

(c)

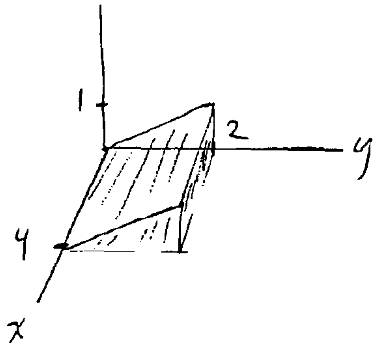


$$\iint_R x\,dA = \int_{y=0}^{3}\int_{x=\frac{4y}{3}}^{\sqrt{25-y^2}} x\,dx\,dy$$

$$= \boxed{25}$$

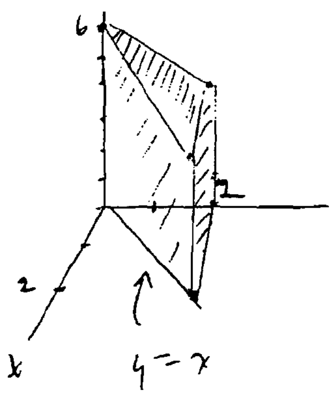
3

(a)



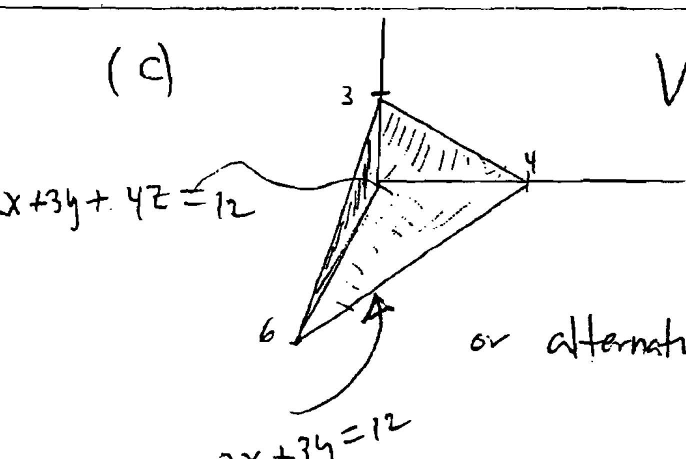
$$V = \iint_R z \, dA = \int_0^4 \int_0^2 \frac{y}{2} \, dy \, dx = 4$$

(b)



$$V = \int_{x=0}^{2} \left[\int_{y=x}^{2} (6 - x - y) \, dy \right] dx = 8$$

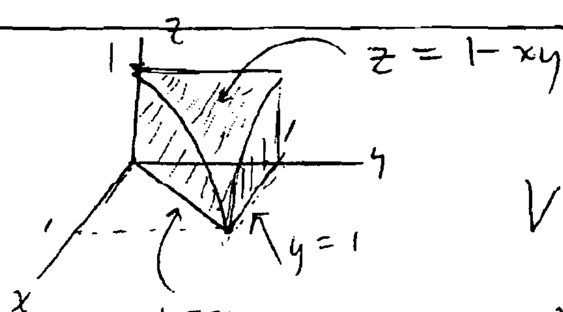
(c)



$$V = \int_{x=0}^{6} \int_{y=0}^{4 - \frac{2x}{3}} \frac{12 - 2x - 3y}{4} \, dy \, dx = 12$$

or alternatively: $$V = \int_{y=0}^{4} \int_{x=0}^{6 - \frac{3}{2}y} \frac{12 - 2x - 3y}{4} \, dx \, dy$$

(d)



$$V = \int_{x=0}^{1} \int_{y=x}^{1} (1 - xy) \, dy \, dx = 3/8$$