# A variety of Vector Problems

1. Recall the triangle inequality for real numbers says:

   for any two real numbers a and b. There is a vector version that says:

   for any two vectors **a** and **b**. Verify this inequality holds for the specific vectors **u** = (-3, 2), **v** = (1, -2). Also verify it for the vectors **u** = (2, 1) and **v** = (5, 4).

2. Find the two unit normal vectors to the graph of the curve $y = x^3$ at the point (1, 1). Repeat for the point (2, 8).

3. Forces with magnitudes of 500 pounds and 200 pounds act on a machine part at angles of 30 degrees and –45 degrees with the positive x-axis. Find the direction and magnitude of the resultant forces.

4. Three forces with magnitudes of 75 pounds, 100 pounds and 125 pounds all act on the same object at angles of 30 degrees, 45 degrees and 120 degrees with the positive x-axis. Find the direction and magnitude of the resultant force.

5. Use vectors to find the points of trisection of the line segment with endpoints (1, 2) and (7, 5).

6. Using vectors, prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and has one half the length of the third side.

7. Determine whether the triangle in space with vertices (0, 0, 0), (2, 2, 1), (2, -4, 4) is a right triangle.

8. Find an equation for the sphere that has center at (-2, 1, 1) and is tangent to the xy-plane.

9. Find the center and radius of the sphere given by:

10. use vectors to determine whether the points lie in a straight line: (0, -2, -5), (3, 4, 4), (2, 2, 1).

11. Find a vector with magnitude exactly ¾ and in the same direction as the vector v = (-4, 6, 2).

12. Use vectors to find the point that lies exactly two-thirds of the way from P to Q, where P=(4, 3, 0) and Q = (1, -3, 3).

# A Variety of Vector Probs – Solutions

(1) $\|\vec{u}+\vec{v}\| = \|(-3,2)+(1,-2)\| = \|(-2,0)\| = \sqrt{(-2)^2+0^2} = 2$

$\|\vec{u}\| = \|(-3,2)\| = \sqrt{(-3)^2+2^2} = \sqrt{13}$

$\|\vec{v}\| = \|(1,-2)\| = \sqrt{1^2+(-2)^2} = \sqrt{5}$

Clearly $\underbrace{\|\vec{u}+\vec{v}\|}_{2} \le \underbrace{\|\vec{u}\|+\|\vec{v}\|}_{5.84\ldots}$ for the given vectors

– You go verify for $\vec{u} = (2,1)$ and $\vec{v} = (5,4)$

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(2) Both unit normal vectors are perpendicular to the tan line at $(1,1)$. $m_{tan}$ at $(1,1)$ is $y'(1) = 3x^2\big|_{x=1} = 3$, so

$m_\perp = -1/3$. Both $\vec{v_1} =: 3i-j$ and $-\vec{v_1} =: \vec{v_2} = -3i+j$ are vectors normal to the tan line at $(1,1)$, unit normals are obtained as $\vec{u_1} =: \dfrac{\vec{v_1}}{\|\vec{v_1}\|} = \dfrac{3i-j}{\sqrt{9+1}} = \dfrac{3}{\sqrt{10}}i - \dfrac{1}{\sqrt{10}}j$

and $\vec{u_2} =: -\dfrac{3}{\sqrt{10}}i + \dfrac{1}{\sqrt{10}}j$

Now you go do it for $(2,8)$, etc.

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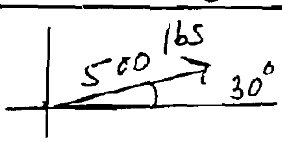
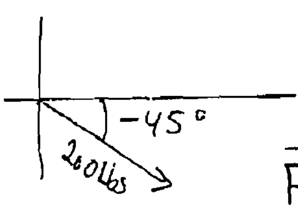
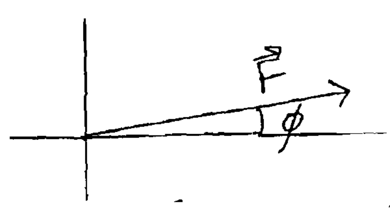
(3)



$\vec{F_1} = 500\cos 30° i + 500 \sin 30° j \doteq 433i + 250j$




$\vec{F_2} = 200\cos(-45°)\, i + 200\sin(-45°)\, j \doteq 141.4i - 141.4j$

$\vec{F} = \vec{F_1} + \vec{F_2} = 574.4i + 108.6j$




$\tan\phi = \dfrac{108.6}{574.4} \Rightarrow \phi \approx 10.7°$

④ $\vec{F_1} = 75\cos 30^\circ i + 75\sin 30^\circ j \doteq 64.95i + 37.5j$

$\vec{F_2} = 70.71i + 70.71j$

$\vec{F_3} = -62.5i + 108.25j$

$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} = 73.16i + 216.46j$

$\tan\phi = \dfrac{216.46}{73.16} \Longrightarrow \phi \approx 71.3^\circ$

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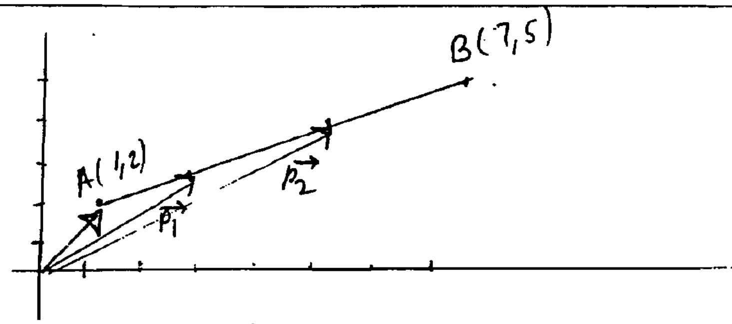
⑤



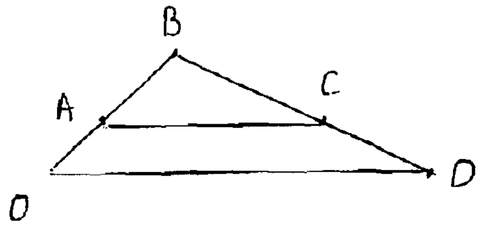

Let $\vec{P_1}$ be the position vector of the point $\frac{1}{3}$ of the way from A to B and $\vec{P_2}$ the position vector of the point $\frac{2}{3}$ of the way from A to B.

$$\vec{P_1} = \vec{OA} + \frac{1}{3}\vec{AB} = (1,2) + \frac{1}{3}[(7,5)-(1,2)] = (1,2) + \frac{1}{3}(6,3)$$
$$= (1,2) + (2,1) = (3,3)$$

$$\vec{P_2} = \vec{OA} + \frac{2}{3}\vec{AB} = (1,2) + (4,2) = (5,4)$$

so the points $(3,3)$ and $(5,4)$ trisect the line.

(6)




Given: $\overrightarrow{OA} = \overrightarrow{AB}$ and $\overrightarrow{BC} = \overrightarrow{CD}$

Show: $2\overrightarrow{AC} = \overrightarrow{OD}$

$$\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = 2\overrightarrow{OA} + 2\overrightarrow{BC} =$$

$$2\overrightarrow{OA} + 2[\overrightarrow{OC} - \overrightarrow{OB}] = 2\overrightarrow{OA} + 2[\overrightarrow{OC} - 2\overrightarrow{OA}]$$

$$= 2\overrightarrow{OA} + 2\overrightarrow{OC} - 4\overrightarrow{OA} =$$

$$\cancel{2\overrightarrow{OA}} + 2\overrightarrow{OC} - 2\overrightarrow{OA} \cancel{- 2\overrightarrow{OA}} =$$

$$2[\overrightarrow{OC} - \overrightarrow{OA}] = 2\overrightarrow{AC}$$

$$\therefore \quad 2\overrightarrow{AC} = \overrightarrow{OD}$$

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(7) $O = (0,0,0)$, $A = (2,2,1)$, $B = (2,-4,4)$

$$\left.\begin{aligned} \overline{OB}^2 &= 2^2+4^2+4^2 = 36 \\ \overline{OA}^2 &= 2^2+2^2+1^2 = 9 \\ \overline{AB}^2 &= 0^2+6^2+3^2 = 45 \end{aligned}\right\} \quad \overline{AB}^2 = \overline{OA}^2 + \overline{OB}^2$$

$\therefore$ $\Delta OAB$ is a right triangle by the converse of Pythagorean theorem.

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(8) $(x+2)^2 + (y-1)^2 + (z-1)^2 = 1$

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(9) Completing square yields $(x-\frac{1}{3})^2 + (y+1)^2 + z^2 = 1$

so center $= (1/3, -1, 0)$, $r = 1$.

(OVER →)

(10) $A = (0,-2,-5)$, $B = (3,4,4)$, $C = (2,2,1)$

$$\left.\begin{array}{l}\overrightarrow{AB} = (3,6,9) \\ \overrightarrow{CB} = (1,2,3)\end{array}\right\} \quad \overrightarrow{AB} = 3\overrightarrow{CB}$$

$\therefore$ $\overrightarrow{CB} \parallel \overrightarrow{AB}$, so all 3 points determine parallel vectors, hence they lie on the same line.

(11)

$$\text{Desired vector} = \frac{3}{4}\frac{\vec{V}}{\|\vec{V}\|} = \frac{3}{4}\frac{(-4,6,2)}{\sqrt{4^2+6^2+2^2}} = \frac{(-3, 9/2, 3/2)}{\sqrt{56}}$$

etc.

(12) For Desired point A (identified with position vector $\overrightarrow{OA}$):

$$\overrightarrow{OA} = \overrightarrow{OP} + \frac{2}{3}[\overrightarrow{OQ} - \overrightarrow{OP}] =$$

$$(4,3,0) + \frac{2}{3}[(-3,-6,3)] = (2,-1,2)$$

So A is the point $(2,-1,2)$.