Various and varied vector problems

1. Find the angle between the vectors u = (1,1) and v = (2, -2).
2. Find the angle between the vectors u = (1, 1, 1) and v = (2, 1, -1)
3. What is known about the smallest angle between two vectors if their dot product is negative? Positive? zero?

4.

Consider the vectors u= $(\cos\alpha, \sin\alpha, 0)$ and v $= (\cos\beta, \sin\beta, 0)$, where $\alpha > \beta$. Find the dot product of the two vectors in two different ways and thereby prove the trigonometric identity:
$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\ \sin\beta$

5. A toy wagon is pulled by exerting a force of 15 lbs on a handle that makes a 30 degree angle with the horizontal. Find the work done in pulling the wagon 50 ft.

6. Find the work done in moving a particle from P to Q if the magnitude and direction of the force are given by the vector **v**:

   P= (1, 3, 0), Q = (-3, 5, 10), v = -2i + 3j + 6k

7. Given the vectors u = (2, 3) and v = (5, 1), find the projection of u onto v and also find the component of u orthogonal to v.

8. A 32,000-pound truck is parked on a 15 degree slope. Assume the only force to overcome is that due to gravity. Find the force required to keep the truck from rolling down the hill. Also find the force perpendicular to the hill..

9. If the projection of **u** onto **v** has the same magnitude as the projection of **v** onto **u**, then is it true that the magnitudes of **u** and **v** are the same?

10. Find a unit vector that is orthogonal to v = 3i+2j-k.

Solutions for Various and Varied Vector Problems

(1) $\vec{u}\cdot\vec{v} = (1,1)\cdot(2,-2) = 2 + -2 = 0$

$\therefore$ $\vec{u}$ and $\vec{v}$ are orthogonal

(2) $\vec{u}\cdot\vec{v} = (1,1,1)\cdot(2,1,-1) = 2+1+-1 = 2$

$\vec{u}\cdot\vec{v} = \|\vec{u}\|\|\vec{v}\|\cos\theta = \sqrt{3}\sqrt{6}\cos\theta$

$\therefore\ 2 = \sqrt{18}\cos\theta$

$\theta = \cos^{-1}(2/\sqrt{18})$

$\theta \approx 61.87^\circ$

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(3) $\vec{u}\cdot\vec{v}$ negative $\Rightarrow \|\vec{u}\|\|\vec{v}\|\cos\theta < 0 \Rightarrow \cos\theta < 0$

$\therefore\ 90^\circ < \theta < 180^\circ$

$\vec{u}\cdot\vec{v}$: positive $\Rightarrow \|\vec{u}\|\|\vec{v}\|\cos\theta > 0 \Rightarrow \cos\theta > 0$

$\therefore\ 0 < \theta < 90^\circ$

$\vec{u}\cdot\vec{v} = 0 \Rightarrow \theta = 90^\circ$

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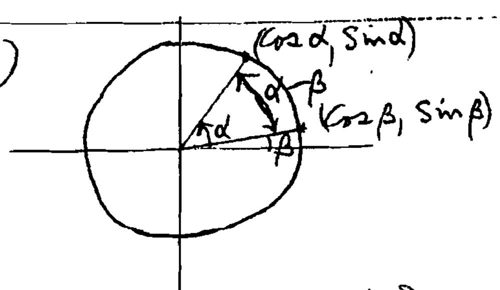
(4)



$\vec{u}\cdot\vec{v} = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Also $\vec{u}\cdot\vec{v} = \underbrace{\|\vec{u}\|}_{1}\,\underbrace{\|\vec{v}\|}_{1}\cos\underbrace{(\alpha-\beta)}_{\text{angle between vectors}}$

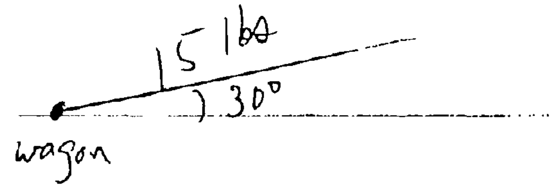
$= \cos(\alpha-\beta)$

$\| = \sqrt{\cos^2\alpha+\sin^2\alpha} = 1 = \|\vec{u}\|$

$\vec{v}\| = \sqrt{\cos^2\beta+\sin^2\beta} = 1 = \|\vec{v}\|$

$\therefore\ \cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha-\beta)$

(5)



$$\text{Work} = (\underbrace{15\cos 30^{\circ}}_{\text{lbs}})(50\text{ ft}) \doteq 649.5 \text{ ft}\cdot\text{lbs}$$

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(6) $\text{work} = \vec{V}\cdot\vec{PQ} = (-2,3,6)\cdot(-4,2,10) = 74$

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(7) $\vec{w} = \text{proj}_{\vec{V}}\vec{u} = \left(\dfrac{\vec{u}\cdot\vec{V}}{\|\vec{V}\|^2}\right)\vec{V} = \dfrac{13}{26}(5,1) = \frac{1}{2}(5,1) = (5/2, 1/2)$

$$W_{\perp} = \vec{u} - \vec{w} = (2,3) - (5/2, 1/2) = (-1/2, 5/2)$$

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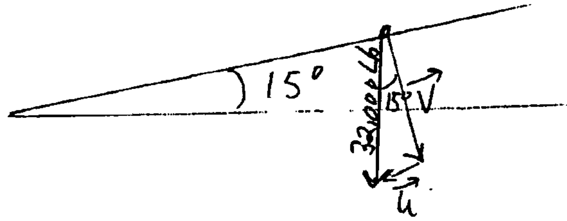
(8)



$$\|\vec{u}\| = 32000 \sin 15^{\circ} \doteq 8282.2 \text{ Lbs}$$

$$\|\vec{V}\| = 32,100 \cos 15^{\circ} \doteq 30909.6 \text{ Lbs}$$

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(9) yes: $\|\text{proj}_{\vec{V}}\vec{u}\| = \dfrac{|\vec{u}\cdot\vec{V}|}{\|\vec{V}\|} = \text{proj}_{\vec{u}}\vec{V} = \dfrac{|\vec{u}\cdot\vec{V}|}{\|\vec{u}\|}$

$$\Longrightarrow \frac{\cancel{|\vec{u}\cdot\vec{V}|}}{\|\vec{V}\|} = \frac{\cancel{|\vec{u}\cdot\vec{V}|}}{\|\vec{u}\|} \Longrightarrow \|\vec{V}\| = \|\vec{u}\|.$$

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(10) $i - 2j - k$ is orthogonal to $\vec{V}$, so normalize to get a u
vector in same direction; $\dfrac{i-2j-k}{\sqrt{1^2+2^2+1^2}} = \boxed{\frac{1}{\sqrt{6}}i - \frac{2}{\sqrt{6}}j - \frac{1}{\sqrt{6}}k}$