Lines and Planes

1. Find a set of parametric equations and also a set of symmetric equations of the line through the given point parallel to the indicated line or vector. For each line, express the direction numbers as integers:

   (a) (0,0,0) $\mathbf{v} = (1, 2, 3)$

   (b) (-2,0,3) $\mathbf{v} = 2i+4j-2k$

   (c) (1,0,1) $x=3+3t,\ y=5-2t,\ z= -7+t$

   (d) (-3,5,4) $(x-1)/3 = (y+1)/-2 = z-3$

2. Find a set of parametric equations and also a set of symmetric equations of the line through the two points (express the direction numbers as integers):

   (a) (5,-3,-2), (-2/3, 2/3, 1)

   (b) (1,0,1), (1, 3, -2)

3. Find a set of parametric equations of the line described:

   (a) The line passes through (2,3,4) and is parallel to the xz-plane and the yz-plane.

   (b) The line passes through (2,3,4) and is perpendicular to the plane given by $3x+2y-z = 6$.

4. Determine whether the following pairs of lines intersect, and if so, find the point of intersection and the cosine of the acute angle between the planes:

   (a) Line #1 is: $x=4t+2,\ y = 3,\ z = -t+1$
   Line #2 is: $x=2s+2,\ y=2s+3,\ z = s+1$

   (b) Line #1 is: $x/3 = (y-2)/-1 = z+1$.
   Line #2 is: $(x-1)/4 = y+2 = (z+3)/-3$

5. Find an equation of the plane passing through the point that is perpendicular to the given vector or line:

   (a) (2,1,2) $\mathbf{n} = \mathbf{i}$

   (b) (3,2,2) $\mathbf{n} = 2\mathbf{i}+3\mathbf{j}-\mathbf{k}$

   (c) (0,0,6) $x=1-t,\ y = 2+t,\ z = 4-2t.$

6. Find an equation of the plane passing through (0,0,0), (1,2,3) and (-2,3,3).

7. Find an equation of the plane containing the lines given by:
   Line #1: $(x-1)/-2 = y-4 = z$ and Line #2: $(x-2)/-3 = (y-1)/4 = (z-2)/-1$.

8. Find an equation of the plane passing through the points (2,2,1) and (-1,1,-1) that is perpendicular to the plane $2x-3y+z = 3$.

9. Determine the angle of intersection of the following pairs of planes:

   (a) $5x-3y+z = 4$ and $x+4y+7z = 1$.

   (b) $x-3y+6z = 4$ and $5x+y-z = 4$.

   (c) $x-5y-z = 1$ and $5x-25y-5z = -3$

10. Mark the intercepts and make a rough sketch of the graph of each of the following planes:

    (a) $4x+2y+6z = 12$

    (b) $2x-y+3z = 4$

    (c) $y+z = 5$

11. Find the intersection point, if any, of the point and the line. Also determine whether the line lies in the plane:

    Plane: $2x-2y+z = 12$
    line: $x-1/2 = (y+3/2)/-1 = (z+1)/2$

12. Find the distance between the point (1,2,3) and the plane $2x+3y+z = 4$.

13. Find the distance between the planes $x-3y+4z = 10$ and $x-3y+4z = 6$.

Solutions for Lines and planes

1) (a) $$\frac{x-0}{1} = \frac{y-0}{2} = \frac{z-0}{3}$$

$$x = y/2 = z/3$$ symmetric form

$$\left.\begin{array}{l} x = t \\ y = 2t \\ z = 3t \end{array}\right\}$$ parametric form

(b) $$\frac{x+2}{2} = \frac{y-0}{4} = \frac{z-3}{-2}$$ Symmetric form

$$\left.\begin{array}{l} x = 2t-2 \\ y = 4t \\ z = -2t+3 \end{array}\right\}$$ Parametric form

(c) $$\frac{x-1}{3} = \frac{y-0}{-2} = \frac{z-1}{1}$$ Symmetric form

$$\left.\begin{array}{l} x = 3t+1 \\ y = -2t \\ z = t+1 \end{array}\right\}$$ parametric form

(d) $$\frac{x+3}{3} = \frac{y-5}{-2} = \frac{z-4}{1}$$ symmetric form.

$$\left.\begin{array}{l} x = 3t-3 \\ y = -2t+5 \\ z = t+4 \end{array}\right\}$$ parametric form

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(2) (a) $\vec{V} = [5-(-2/3)]i + (-3-2/3)j + (-2-1)k$

$= \frac{17}{3}i - \frac{11}{3}j - 3k$

$$\frac{x-5}{17/3} = \frac{y+3}{-11/3} = \frac{z+2}{-3}$$

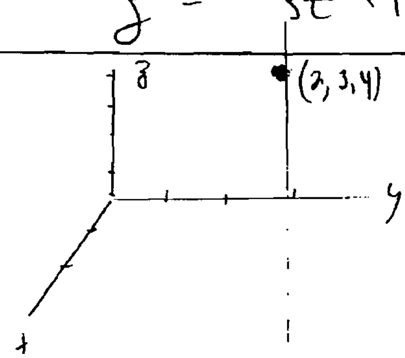
or $$\frac{x-5}{17} = \frac{y+3}{-11} = \frac{z+2}{-9}$$ symmetric form

$$\left.\begin{aligned} x &= 17t+5 \\ y &= -11t-3 \\ z &= -9t-2 \end{aligned}\right\}$$ parametric form.

(b) $\vec{V} = (0, 3, -3)$, so line is parallel to yz-plane, so no symmetric form. parametric form is:

$$\left.\begin{aligned} x &= 1 \\ y &= 3t+0 \\ z &= -3t+1 \end{aligned}\right\}$$

(3) (a)



$$\left.\begin{aligned} x &= 2 \\ y &= 3 \\ z &= t \end{aligned}\right\}$$

(b) $$\left.\begin{aligned} x &= 2+3t \\ y &= 3+2t \\ z &= 4+-t \end{aligned}\right\}$$

4 (a) $4t + 2 = 2s + 2 \Rightarrow s = 2t$

so $3 = 2s + 3 \Rightarrow s = 0$

$\therefore t = \frac{1}{2}s = \frac{1}{2}(0) = 0$

so for Line#1 : $\left.\begin{array}{l} x = 4(0)+2 = 2 \\ y = 3 \\ z = -0+1 = 1 \end{array}\right\}$

for L_2 : $\left.\begin{array}{l} x = 2(0)+2 = 2 \\ y = 2(0)+3 = 3 \\ z = 0+1 = 1 \end{array}\right\}$

$\therefore$ (2, 3, 1) is point of intersection

$\vec{V_1} = (4, 0, -1)$, $\vec{V_2} = (2, 2, 1)$

$\vec{V_1} \cdot \vec{V_2} = 7 = \sqrt{17}\sqrt{9}\cos\theta$

$7 = 3\sqrt{17}\cos\theta$

$\cos\theta = \frac{7}{3\sqrt{17}}$, etc.

(b) $x = 3t$, $y = -t+2$, $z = t-1$ for Line#1 (L_1)

$x = 4s+1$, $y = s-2$, $z = -3s-3$ for Line #2 (L_2)

$3t = 4s+1 \Rightarrow 3t - 4s = 1$
$-t+2 = s-2 \Rightarrow t + s = 4$

$\Rightarrow \left.\begin{array}{l} 3t - 4s = 1 \\ 4t + 4s = 16 \end{array}\right\}$

$7t = 17$

$t = 17/7$

Then for L_1: $x = 3(17/7) = 51/7$
for L_2: $x = 4(\frac{17}{7}) + 1 = 75/7$

Contradiction!!
$\therefore$ Lines don't intersect.

(5) (a) $1(x-2) + 0\,(y-1) + 0\,(z-2) = 0$

$\therefore x = 2$

(b) $2(x-3) + 3(y-2) + -1(z-2) = 0$, etc

(c) $-1(x-0) + 1(y-0) + -2(z-6) = 0$, etc

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(6) $A(0,0,0)$
$B(1,2,3)$
$C(-2,3,3)$

$\vec{N} = \vec{AB} \times \vec{AC} = (1,2,3) \times (-2,3,3)$
$= (-3,-9,7)$

so equation of plane is $-3(x-0) + -9(y-0) + 7(z-0) = 0$

or simply $3x + 9y - 7z = 0$

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(7) Let $\vec{N} = (-3,4,-1) \times (-2,1,1) = (5,5,5)$

Then $\vec{N}$ is normal to desired plane. Note $(1,4,0)$ lies on a line in desired plane, so $(1,4,0)$ lies in desired plane, so its equation is:

$5(x-1) + 5(y-4) + 5(z-0) = 0$ or

Simply $x + y + z = 5$

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(8) $\vec{v}_1 = (2,-3,1)$ is parallel to desired plane

$\vec{v}_2 = (2,2,1) - (-1,1,-1) = (3,1,2)$ is also parallel to desired plane.

Let $\vec{N} = \vec{v}_1 \times \vec{v}_2 = (-7,-1,11)$: normal to desired plane.

so Eq. of desired plane is $-7(x+1) + -1(y-1) + 11(z+1) =$

or simply $-7x - y + 11z = -5$

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9 (a) $\vec{V_1} \cdot \vec{V_2} = (5,-3,1) \cdot (1,4,7) = 0 \quad \therefore \theta = 90^\circ$

(b) $\vec{V_1} \cdot \vec{V_2} = (1,-3,6) \cdot (5,1,-1) = -4$

$-4 = \sqrt{1+9+36}\ \sqrt{25+1+1}\ \cos\theta$

$\theta = \cos^{-1}\left(-4/\sqrt{1242}\right) \doteq 96.52^\circ$

(c) $\vec{V_1} \cdot \vec{V_2} = (1,-5,-1) \cdot (5,-25,-5) = 135$

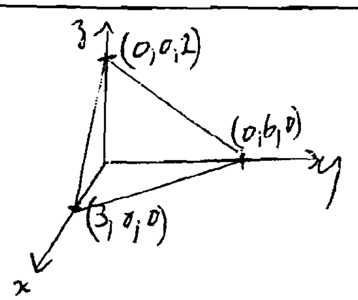
$135 = \sqrt{1+25+1}\ \sqrt{25+625+25}\ \cos\theta$

$135 = \sqrt{18225}\ \cos\theta$

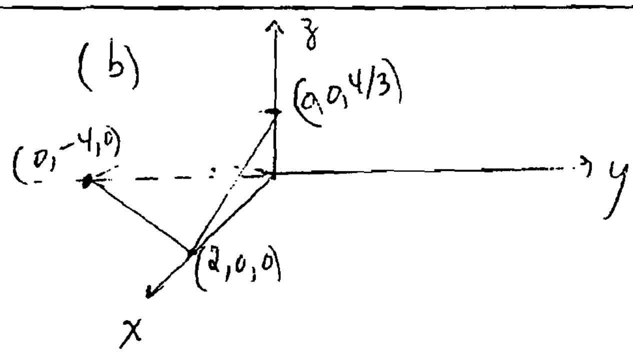
$\theta \approx \cos^{-1}\left(135/\sqrt{18225}\right) = \cos^{-1}(1) = 0$

(so planes are parallel)

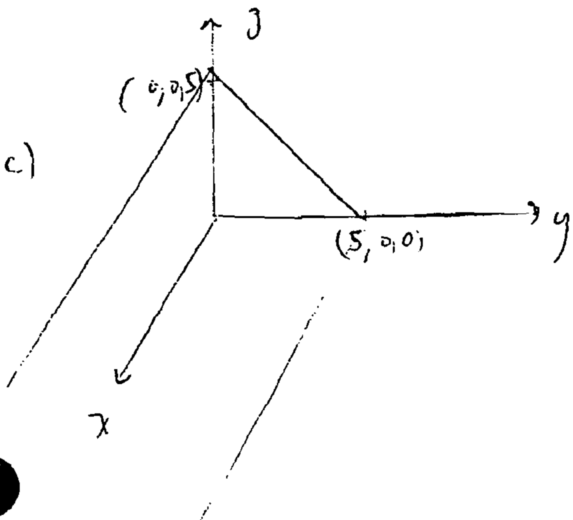
10 (a)



(b)



(c)



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(11) parametric form of line: $\left.\begin{aligned} x &= t + 1/2 \\ y &= -t - 3/2 \\ z &= 2t - 1 \end{aligned}\right\}$

$$2(t + 1/2) - 2(-t - 3/2) + 2t - 1 = 12$$

$\vdots$

$$t = 3/2$$

then $\left.\begin{aligned} x &= 3/2 + 1/2 = 2 \\ y &= -3/2 - 3/2 = -3 \\ z &= 2(3/2) - 1 = 2 \end{aligned}\right\}$ so $(2, -3, 2)$ is point

of intersection. The line does not lie in the given plane since, for example, when $t = 1/2$, then $(1, -2, 0)$ lies on the line but $2(1) - 2(-2) + 1(0) \neq 12$, so $(1, -2, 0)$ does not lie in given plane.

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(12) $P(1,2,3)$. Note $Q(2,0,0)$ lies in given plane and $\vec{N} = (2,3,1)$ is perp. to given plane. So: $(1,-2,-3)$

$$D = \|\text{proj}_{\vec{N}} \overrightarrow{PQ}\| = \frac{|\overrightarrow{PQ} \cdot \vec{N}|}{\|\vec{N}\|} = 7/\sqrt{14}\text{, etc.}$$

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(13) The point $A(6,0,0)$ is in one plane and $B(10,0,0)$ lies in the other. $\vec{N} = (1,-3,4)$ is perp. to both planes.

$$D = \|\text{proj}_{\vec{N}} \overrightarrow{AB}\| = \frac{|\overrightarrow{AB} \cdot \vec{N}|}{\vec{N}} = 4/\sqrt{26}\text{, etc.}$$