Note: $T^2 = \frac{4\pi^2 a^3}{GM}$, where T = period, a = semi-major axis

Application of Vectors to Velocity and Acceleration

1. The Viking 2 Orbiter, which surveyed Mars from September 1975 to August 1976, moved in an elliptical orbit around Mars. The semi-major axis of this orbit was 22, 030 km. Show the orbital period was about 1655 minutes.
   Hint: The gravity of Mars holds the Orbiter in its elliptical orbit about Mars in exactly the same way that the sun holds each planet in its orbit about the sun. Assume the following constant values:

   Newton's Constant of Universal Gravitation $G = 6.6720 \times 10^{-11} Nm^2 kg^{-2}$
   When using this constant, all distances must be expressed in meters, mass in kilograms and time in seconds.

   Mass of Mars is $6.418 \times 10^{23}\ kg$

2. In July 1965, the USSR launched Proton I, weighing 12,200 kg (at launch), with a perigee (minimum distance above earth) of 183 km and an apogee of 589 km. The period of the orbit of Proton I was 92.25 minutes. Use this information in conjunction with any relevant information presented in lecture about orbits of objects (determined by the force of gravitational attraction) to compute the radius of the earth. Give your answer in kilometers. Then go look up the radius of the earth (site the reference, please) and compare your calculated result with this value. Note:

   Earth's Mass $= 5.975 \times 10^{24}\ kg$

3. A particle moves so that its derivative of its position vector is always orthogonal to the position vector. Show that the particle moves on a circle.

4. Determine the maximum height and range of a projectile fired 1.5 m above the ground with an initial velocity of 100 m/s at an angle of 30 degrees above the horizontal.

5. A projectile is fired from ground level at an angle of 8 degrees with the horizontal. Find the smallest velocity necessary if the projectile is to have a range of 50 meters.

6L

Application of Vectors to Vel. and Accel. – Solutions

(1) use $T^2 = \dfrac{4\pi^2 a^3}{GM}$, where a = semi-major Axis.

$a = 22030 \times 10^3$ m, so $T = \sqrt{\dfrac{4\pi^2 a^3}{GM}}$

$$= 2\pi a \sqrt{\frac{a}{GM}} = 2\pi(22030 \times 10^3)\sqrt{\frac{22030 \times 10^3}{6.6720 \times 10^{-11} \times 6.418 \times 10^{23}}}$$

$$\doteq 99283 \text{ sec} \doteq 1655 \text{ min.}$$

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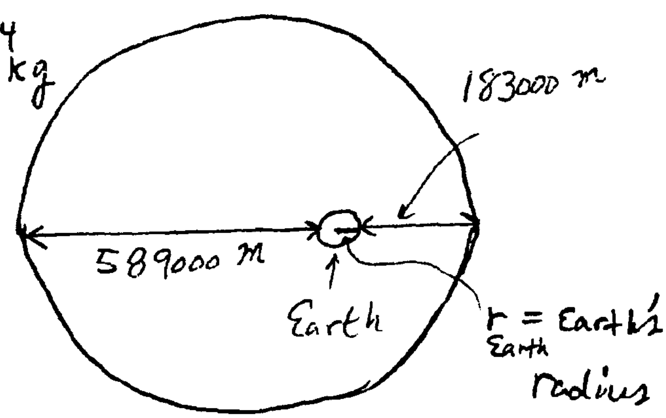
(2) From $T^2 = \dfrac{4\pi^2 a^3}{GM}$, $a = \left(\dfrac{T^2 GM}{4\pi^2}\right)^{1/3}$

where M = Earth's mass $= 5.975 \times 10^{24}$ kg

92.25 min = 5535 sec



so $a = \left(\dfrac{5535^2 \cdot 6.672 \times 10^{-11} \times 5.975 \times 10^{24}}{4\pi^2}\right)^{1/3}$

$$\doteq 6763266.28 \text{ m}$$

Now major Axis $= 2a = 2(6763266.28) = 589000 + 183000 + 2r_{Earth}$

$\Longrightarrow r_{Earth} = 6377266.28$ m $= 3,962.6$ miles

(OVER →)

(3) Let $\vec{r}(t)$ = position vector. We are given $\vec{r} \cdot \vec{r}\,' = 0$ at every time t.

We must show $\|\vec{r}\|$ is Constant.

Now $(\vec{r} \cdot \vec{r})' = \vec{r} \cdot \vec{r}\,' + \vec{r}\,' \cdot \vec{r} = 2\vec{r} \cdot \vec{r}\,' = 0$.

$\therefore$ $\vec{r} \cdot \vec{r}$ is Constant

since $\vec{r} \cdot \vec{r} = \|\vec{r}\|^2$, $\|\vec{r}\|^2$ and hence $\|\vec{r}\|$ is Constant.

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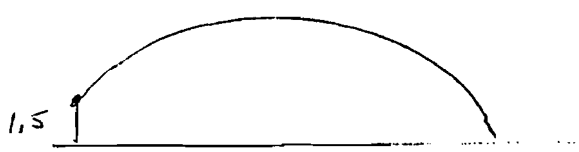
(4)



$V_0 = 100$ m/s

$x = 100 \cos 30^\circ t = 50\sqrt{3}\,t$

$$y = 1.5 - \tfrac{1}{2}(9.8)t^2 + 100 \sin 30^\circ t$$

$$y = 1.5 - 4.9t^2 + 50t$$

$y' = 0 \Rightarrow -9.8t + 50 = 0 \Rightarrow t \doteq 5.1$ sec

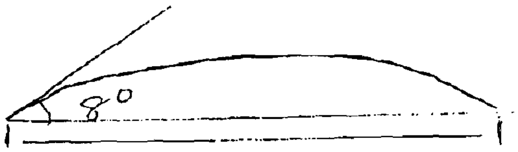
then $y(5.1) \doteq 129$ m

$y = 0$ when $t \doteq 10.23$ sec

Then $x(10.23) \doteq 886$ m

(5)

$V_0 = 100$ m/s



$$x = V_0 \cos 8^\circ\, t$$

$$y = \frac{1}{2}(-9.8)t^2 + V_0 \sin 8^\circ t = -4.9t^2 + V_0 \sin 8^\circ t$$

$$50 = V_0 \cos 8^\circ t \text{ when } t = \frac{50}{V_0 \cos 8^\circ}$$

$$\text{so} \quad -4.9\left(\frac{50}{V_0 \cos 8^\circ}\right)^2 + V_0 \sin 8^\circ \left(\frac{50}{V_0 \cos 8^\circ}\right) = 0$$

$$\Rightarrow v_0 \doteq 42.2 \text{ m/s}$$