Fall 1998
Math 152, Sections 510–512
Final
Version 1

Name:

Student Number:

Section Number:

Show all working.
Write your final answer in the space provided.
Calculators are permitted.
Time available: 2 hours

1. (a) Find the area under the curve

$$y = \frac{x^2}{\sqrt{1-x^2}}$$

on the interval $[0, 1]$.

(b) Evaluate the integral

$$\int \frac{3x-2}{(x-1)(x+1)^2}\, dx.$$

2. A spherical water tower has a radius of 4m and the bottom of the sphere is 20m above the ground. Calculate the work required to completely fill the tank, assuming the water starts at ground level.

(Density of water = 1000kg/m^3, acceleration due to gravity = 9.8m/s^2)

3. Find the Taylor series expansion of $f(x) = \ln x$ about $x = 1$. Find the radius of convergence of the Taylor series.

4. Solve the initial value problem

$$y' + 2xy = 2x^5$$

where $y(0) = 0$.

5. Use Simpson's rule with $n = 6$ to approximate

$$\int_0^1 \cos x^2 \, dx.$$

Estimate the error.

6. Write

$$\int_0^1 \cos x^2 \, dx$$

as an infinite series. Use the first 3 terms to estimate the value of the integral, and estimate the error.

7. Find the angle between the planes

$$x + 4y - 3z = 1 \qquad \text{and} \qquad -3x + 6y + 7z = 0.$$

Find the line of intersection and write down its parametric form.

8. Find the length of the curve

$$x = e^t \cos t, \qquad y = e^t \sin t$$

on the interval $[0, \pi]$.

9. Let $\mathbf{r}(t) = \langle \sin t, \cos t, t^2 \rangle$. Find the unit tangent vector, unit normal vector and binormal vector.

10. An aeroplane has velocity vector $\mathbf{v}(t) = \langle 1, 3t^2 - 1, -\frac{2t}{(1+t^2)^2} \rangle$ and passes throught the point $(0, 0, 1)$. Find the displacement and acceleration of the plane at time t.

Bonus Question

(a) If you needed to estimate

$$\int_0^1 \cos x^2 \, dx$$

to 10 decimal places, which method would you prefer - Simpson's rule or Maclaurin series? Give reasons backed by mathematical reasoning.

(b) The area under a function $y = f(x)$ on an interval $[a, b]$ is rotated about the y-axis. Write down the volume of the solid of revolution so formed. Write down the centre of mass. Show that the theorem of Pappus holds true for this solid.