Final

Math 181, Section 5

December 15th, 1999

Name:

Calculators are permitted unless thay have a built-in algebra system.
You are permitted one two-sided letter-sized sheet of handwritten notes.

At least one mark will be taken off for every constant of integration that is missing!

Part I - Short Answer

Write your answer in the space provided. No partial credit.

Question 1

Write down the following derivatives:

(a) $\dfrac{d}{dx}(7x^3 + 5x^2 + 2 + 1/x)$

(b) $\dfrac{d}{dt}(e^{2t})$

(c) $\dfrac{d}{dx}(x^2 \sec x)$

(d) $\dfrac{d}{d\theta}\left(\dfrac{\sin\theta}{\theta}\right)$

(8 points)

Question 2

Find the value of the following integrals:

(a) $\displaystyle\int_0^{\pi} x \sin(1 - x^2)\ dx$

(b) $\displaystyle\int_0^{e} \frac{1}{x}\ dx$

(c) $\displaystyle\int \cos^2\theta\ d\theta$

(6 points)

Question 3

Write down the following limits:

(a) $\lim_{x\to 3} \frac{4x^2}{x+1} - \cos(\pi x)$

(b) $\lim_{t\to\infty} \frac{3t^2 - 5t + 3}{7t^2 + 5t - 1}$

(c) $\lim_{h\to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$

(6 points)

Question 4

Differentiate

$$y = \frac{(3x+5)^{12}\sqrt{8x^2+1}}{(10x+5)^3}.$$

(Hint: if you don't use logarithmic differentiation this will take you a long time to do)

(8 points)

Question 5

A thin rod running from $x = 0$ to $x = 2\pi$ has linear density $\delta(x) = \sin x + 2$. Find the centre of mass.

(4 points)

Question 6

Consider the function

$$f(x) = \cos x - x$$

on the interval $[0, \pi]$.

Which of the following statements are false, and which are true:

a. $f(x)$ is continuous.

b. $f(x) = 0$ at some point in $[0, \pi]$.

c. $\int f(x)\ dx = \sin x - x^2/2$.

d. $f'(x) = -\frac{\pi+2}{\pi}$ at some point in $[0, \pi]$.

e. $f'(x) = 0$ at some point in $[0, \pi]$.

(5 points)

Question 7

The curve $f(x) = \sin x$ is rotated about the x-axis over the interval $[0, \pi/2]$. Write down, but do not attempt to integrate, the integral which gives the surface area swept out by the curve.

(5 points)

Question 8

Find the absolute maximum and minimum values of the function

$$f(x) = 2x^3 + 3x^2 - 12x + 1$$

on the interval $[-4, 2]$.

(8 points)

Part II - Long Answer

You must show all relevant working. You will get no credit for a correct answer if there is no working.

Question 9

A company is laying pipeline from an on-shore refinery to an offshore oil platform, as illustrated here:

If it costs 10000 to lay a kilometer of pipeline on land, and 50000 to lay a kilometer of pipeline underwater, find the minimum cost to lay the pipeline.

(15 points)

Question 10

A dam on a farm is the shape of a hemisphere with a radius of 10 m embeded in the ground. The farmer wants to pump all the water out of it. Given that water has a mass density of 1000kgm^{-3}, how much work will be done pumping the water out?

(15 points)

Question 11

Graph the function

$$f(x) = 2x^3 + 3x^2 - 12x + 1.$$

Be sure to indicate where it is increasing and decreasing, concave up and concave down, and where any critical points and inflection points may be.

(10 points)

Question 12

Verfiy, using the limit definition of the derivative, that for any functions $f(x)$ and $g(x)$,

$$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x).$$

(10 points)

Extra Credit

You must show all relevant working. If you will get no credit for a correct answer if there is no working.

Question 13

By calculating Riemann sums, show that

$$\int_0^a x^2 \; dx = \frac{1}{3}(a^3).$$

(20 points)