Midterm 2

Question 1

(a) $12/5x^5 - 3/4x^4 + 7/3x^3 + 4x + C$
(b) $x/2 - 1/4\sin(2x) + C$
(c) $\sqrt{1+t^2} + C$

Question 2

(a) 479/60.
(b) 1.
(c) 10/3

Question 3

The radius at at x is $\sqrt{16-x^2}$ so the area $A(x) = \pi(16-x^2)$. Evalute at -3, -1, 1 and three, with disks of width 2: $\pi(7+15+15+7)2 = 88\pi$.

Question 4

(a) 3/2.
(b) 0.
(c) 0.

Question 5

$-10\pi^2\sin(t/\pi) + 10$.

Question 6

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt} = 100\pi$$

Question 7

(a) $C = \pi rl + 4\pi r^2$
(b) Need to get rid of one of r or l:

$$V = 1 = \frac{\pi r^2 l}{2}$$

so

$$l = \frac{2}{\pi r^2}$$

and

$$C = \frac{2}{r} + 4\pi r^2$$

Also r and l must be positive, so $r \in (0,\infty)$. Differentiating:

$$C' = -\frac{2}{r^2} + 8\pi r$$

Now set to zero and solve:

$$r = \frac{1}{\sqrt[3]{4\pi}}.$$

Testing the cost at this point and as $r \to 0$ and $r \to \infty$, we see the minimum C is at

$$r = \frac{1}{\sqrt[3]{4\pi}}.$$

For this r,

$$l = \frac{1}{2^{1/3}\pi^{5/3}}$$

Question 8

(a) Using polynomial long-division

$$f(x) = 4x^2 - 8x + \frac{1}{x-1}$$

so the dominant term is

$$4x^2 - 8x.$$

No horizontal asymptotes. Vertical asymptote at $x = 1$.
(b) Differentiating:

$$f'(x) = 8x - 8 - \frac{1}{(x-1)^2}$$

Setting equal to zero, there is a critical point at $3/2$.
Testing intervals gives:
$(-\infty, 1)$: Decreasing
$(1, 3/2)$: Decreasing
$(3/2, \infty)$: Increasing
So $x = 3/2$ is a local minimum.
(c) Differentiating:

$$f''(x) = 8 + \frac{2}{(x-1)^3}$$

Setting to zero and solving gives

$$x = 1 - \frac{1}{\sqrt[3]{4}}$$

Testing intervals gives:
$(-\infty, 1 - \frac{1}{\sqrt[3]{4}})$: Concave up
$(1 - \frac{1}{\sqrt[3]{4}}, 1)$: Concave down
$(1, \infty)$: Concave up
So

$$x = 1 - \frac{1}{\sqrt[3]{4}}$$

is an inflexion point.
(d)

Question 9

(a)

$$f'(x) = 3ax^2 + 2bx + c$$

so critical points are (use quadratic formula):

$$x = \frac{-2b \pm \sqrt{4b^2 - 12ac}}{6a}$$

(b) According to Rolle's Theorem, between any two zeros there is a critical point. If there were 4 or more zeros, then there would have to be three or more critical points, but part (a) shows that there is at most 2.

So we must have at wost three zeros.