

Midterm I

Math 181, Section 5
October 1st, 1999

Name:

Student ID:

Calculators are permitted unless they have a built-in algebra system.
You are permitted one two-sided letter-sized sheet of handwritten notes.

Part I - Short Answer

Write your answer in the space provided. No partial credit.

Question 1

Consider the function $f(x)$ graphed below:

Do the following exist? If so, write down the value.

(a) $\lim_{x \rightarrow 1} f(x)$

(b) $\lim_{x \rightarrow 3^-} f(x)$

(c) $\lim_{x \rightarrow -1} f(x)$

Is $f(x)$ continuous at $x = 2$?

Is $f(x)$ differentiable at $x = 2$?

(10 points)

Question 2

Write down the following limits:

(a) $\lim_{x \rightarrow 3} 4x^2 - 3 \sin(\pi x)$

(b) $\lim_{t \rightarrow 0} \frac{t}{\sin t}$

(c) $\lim_{h \rightarrow 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$

(9 points)

Question 3

Write down the following derivatives:

(a) $\frac{d}{dx}(12x^4 - 3x^3 + 7x^2 - 3x + 4)$

(b) $\frac{d}{dx}(\sin 2x)$

(c) $\frac{d}{dx}(\sec x)$

(12 points)

Question 4

Consider the curve given by $2xy - 4x^2 + 2y^3 = 0$. Find the slope of the tangent curve at the point $(1, 1)$.

(5 points)

Question 5

Consider the function

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & x \neq 0 \\ 12 & x = 0. \end{cases}$$

The limit

$$\lim_{x \rightarrow 0} f(x)$$

- a. Does not exist, because $f(x)$ oscillates wildly.
- b. Does not exist, because although the left-hand and right-hand limits exist, they are not equal.
- c. Does exist and is equal to 0, because the Sandwich Theorem applies with $-x^2 \leq f(x) \leq x^2$.
- d. Does exist and is equal to 12, because $f(0) = 12$.
- e. None of the above.

(5 points)

Question 6

Suppose that functions $f(x)$ and $g(x)$ and their first derivatives have the following values at 0 and 1.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	5	1/3
1	3	-4	-1/3	-8/3

Write down the first derivatives of the following combinations at the given values of x :

(a) $5f(x) + g(x)$, $x = 1$

(b) $f(x)g(x)$, $x = 0$

(c) $f(g(x))$, $x = 0$

(9 points)

Part II - Long Answer

You must show all relevant working. If you will get no credit for a correct answer if there is no working.

Question 7

Consider the function $f(x) = \sqrt{x+1}$. Using limit techniques, find the derivative of the function at $x = 1$.

Verify your answer using the rules of differentiation.

Find the tangent and normal to the graph of the function at this point.

(15 points)

Question 8

Find the derivative of

$$h(t) = \left(\frac{(x-1)^2}{x+1} \right)^{1/2} .$$

(20 points)

Question 9

Using limit techniques, show that if f and g are differentiable functions, then

$$\frac{d}{dx}(f + g) = \frac{df}{dx} + \frac{dg}{dx}.$$

(15 points)