

Midterm II

Math 181, Section 5
November 5th, 1999

Name:

Calculators are permitted unless they have a built-in algebra system.
You are permitted one two-sided letter-sized sheet of handwritten notes.

Part I - Short Answer

Write your answer in the space provided. No partial credit.

Question 1

Find the following indefinite integrals

(a) $\int 12x^4 - 3x^3 + 7x^2 + 4 dx$

(b) $\int \sin^2 \theta d\theta$

(c) $\int \frac{t dt}{\sqrt{t^2 + 1}}$

(9 points)

Question 2

Find the value of the following definite integrals:

(a) $\int_0^1 12x^4 - 3x^3 + 7x^2 + 4 dx$

(b) $\int_0^{\pi/3} \sec u \tan u du$

(c) $\int_0^1 \frac{10\sqrt{v}}{(1 + v^{3/2})^2} dv$

(12 points)

Question 3

Estimate the volume of a sphere of radius 4 by approximating with four subintervals, evaluating at the mid-point of each interval to get the radius of the disk over each interval.

(4 points)

Question 4

Write down the following limits, if they exist:

(a) $\lim_{x \rightarrow \infty} \frac{3x^2 - 4x + 1}{2x^2 - 4} + \frac{1}{x}$

(b) $\lim_{x \rightarrow -\infty} \frac{3}{\sqrt[3]{x}}$

(c) $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$

(9 points)

Question 5

The acceleration of a mass on a spring at time t is given by

$$a(t) = 10 \sin(t/\pi)$$

If the spring had an initial displacement of 10 and an initial velocity of -10π , find the formula for the displacement at time t .

(8 points)

Question 6

A circular plate of metal is heated. When the radius is 500mm, the radius is increasing at a rate of 0.1 mm per minute. At what rate is the plate's area increasing at that instant?

(8 points)

Part II - Long Answer

You must show all relevant working. If you will get no credit for a correct answer if there is no working.

Question 7

A company wants to make a feeding trough with a 1 cubic meter volume. As depicted, the trough is an open half-cylinder. The sides of the trough are made from metal which costs \$1 per square metre, but the semicircular ends need to be much thicker and cost \$8 per square metre.

(a) Write down the cost of materials as a function of the radius of the ends and the length of the trough.

(b) Find the values of the radius and the length which minimise the cost of materials.

(15 points)

Question 9

A general cubic polynomial has formula

$$f(x) = ax^3 + bx^2 + cx + d.$$

(a) Differentiate $f(x)$ and find its critical points.

(b) Explain, using Rolle's theorem, why $f(x)$ can have at most 3 zeroes.

(15 points)

Extra Credit

You must show all relevant working. If you will get no credit for a correct answer if there is no working.

Question 10

Find the value of the definite integral

$$\int_{-1}^{\pi/4} \sqrt{1-x^2} dx.$$

(Hint: sketch the region represented by the integral and find its area as a combination of a sector of a circle, and a triangle)

Using the same idea, find

$$\int_{-1}^a \sqrt{1-x^2} dx.$$

(20 points)