Midterm III

Math 181, Section 5
December 3rd, 1999

Name:

Calculators are permitted unless thay have a built-in algebra system.
You are permitted one two-sided letter-sized sheet of handwritten notes.

At least one mark will be taken off for every constant of integration that is missing!

Part I - Short Answer

Write your answer in the space provided. No partial credit.

Question 1

Given

$$F(x) = \int_1^x t \sin t \; dt,$$

find $\frac{dF}{dx}$.

(8 points)

Question 2

Find the value of the following definite integrals:

(a) $\displaystyle\int_{-1}^{1} 2x \sin(1 - x^2) \; dx$

(b) $\displaystyle\int_0^1 \frac{10\sqrt{v}}{(1 + v^{3/2})^2} \; dv$

(12 points)

Question 3

Write down, but do not attempt to caulculate, the integral which gives the length of the arc of the graph of $y = \sin x$ over the interval $[0, \pi/2]$.

(5 points)

Question 4

Find the area between the graph of $y = \cos^2 x$ and the x axis over the interval $[-\pi/2, \pi/2]$.

(8 points)

Question 5

A thin rod running from $x = 0$ to $x = 2$ has linear density $\delta(x) = 1 + x$. Find the centre of mass.

(8 points)

Question 6

Consider the area between the curves $y = x^2$ and $y = -x$ over the interval from $x = 0$ to $x = 3$. If you were to revolve it about the y-axis, which of the three methods (disc, washer, and cylindrical shells) could you use to calculate the volume? If it is possible, how many integrals would you need to calculate?

Discs:

Washers:

Shells:

(9 points)

Part II - Long Answer

You must show all relevant working. You will get no credit for a correct answer if there is no working.

Question 7

Consider the region between the curve $y = x^2$, the x-axis, and the line $x = 2$.

(a) Find the volume obtained by revolving it about the y-axis.

(b) Find the area of the surface of revolution swept out by the curve $y = x^2$ over the same interval.

(15 points)

Question 8

Putting a satellite in orbit.

The gravitational force of a planet with mass M, acting upon a satellite of mass m at a distance x from the centre of the planet is given by

$$F(x) = -G\frac{Mm}{x^2}$$

where $G = 6.6720 \times 10^{-11}$ $\mathrm{Nm^2kg^{-2}}$ is the universal gravitational constant. (Note that the force is negative, because it is in the opposite direction to increasing x.)

(a) Write down the integral that gives the work done in lifting a satellite from $x = a$ to $x = b$. (Consider only the work done lifting the satellite itself, ignore the work required to lift fuel, rockets, etc.)

(b) A satellite with a mass of 100 kg is launched from the surface of the earth and put into an orbit 1000 km (= 1000000 m) above the surface of the earth. If the radius of the earth is 6370000 m, find the work requred to put the satellite in orbit.

(20 points)

Question 9

Verfiy, using substitution, that for any function $f(x)$,

$$\int_a^b f(x+c)\ dx = \int_{a-c}^{b-c} f(x)\ dx.$$

(15 points)

Extra Credit

You must show all relevant working. If you will get no credit for a correct answer if there is no working.

Question 10

By calculating Riemann sums, show that

$$\int_a^b x^2 \, dx = \frac{1}{3}(b^3 - a^3).$$

(20 points)