Midterm I

Math 182, Section 1

Sample Problems

Question

Integrate

$$\int_{\pi/4}^{\pi/3} \sin^5\theta \cos^3\theta \ d\theta.$$

Question

Integrate

$$\int \sin 5\theta \cos 3\theta \ d\theta.$$

Question

Integrate

$$\int x \ln x \ dx.$$

Question

Integrate

$$\int_1^e x^n \ln x \ dx.$$

for integers $n > 0$.

Question

Integrate

$$\int \frac{1}{\sqrt{x^2+4x}} \ dx.$$

Question

Integrate

$$\int \frac{x+4}{\sqrt{x^2+4x}} \ dx.$$

Question

Integrate

$$\int \frac{x}{\sqrt{x^2+4x}}\ dx.$$

Question

Integrate

$$\int \frac{x+1}{x^2+4x}\ dx.$$

Question

Decompose

$$\frac{x^3+1}{x^3-1}$$

into partial fractions.

Question

Integrate

$$\int \frac{x^6}{x^3-1}\ dx.$$

Question

Integrate

$$\int \frac{\sqrt{x}+1}{x+4\sqrt{x}}\ dx.$$

Question

Which of the following is it possible to integrate in terms of elementary functions?

$$\int e^{x^2}\ dx.$$

$$\int x\sin x\ dx.$$

$$\int \frac{\sin x}{x}\ dx.$$

Question

Show that

$$\int \sin^n x \; dx = -\frac{1}{n} \cos x \sin^{n-1} x + \frac{n-1}{n} \int \sin^{n-2} x \; dx$$

and use it to find

$$\int_0^{\pi/4} \sin^8 x \; dx$$

Question

An ellipse with axes of length $2a$ and $2b$ is given by the formula

$$y = \pm \frac{b}{a} \sqrt{a^2 - x^2}.$$

Find

$$\int_{-a}^{a} \frac{b}{a} \sqrt{a^2 - x^2} \; dx$$

and use this to show that the area of the ellipse is πab.

Question

Recall that

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \sinh x = \frac{e^x - e^{-x}}{2},$$

and that

$$\frac{d}{dx} \cosh x = \sinh x \qquad \frac{d}{dx} \sinh x = \cosh x.$$

Show that if $x = a \sinh t$, then

$$t = \ln(x + \sqrt{x^2 + 1}) - \ln a$$

Verify that

$$\cosh^2 x - \sinh^2 x = 1.$$

Use the substitution $x = a \sinh t$ together with this identity to evaluate the integral

$$\int \sqrt{x^2 + a^2} \; dx.$$

Use a trigonometric substitution and integration by parts, to evaluate

$$\int \sqrt{x^2 + a^2} \; dx.$$

Do the answers agree?

Question

Page 538, Problem 4.

Question

The compression scheme used by the JPEG image format (commonly used on the world wide web) uses what is called the cosine transform. This transform relies on the properties of integrals of the form

$$\int_0^{2\pi} \cos mx \cos nx \; dx$$

where m and n are non-negative integers. Evaluate the integral. (Hint: there are two cases, $m = n$ and $m \neq n$)

Other Questions

Try the Chapter 7 review, pp 534–5, exercises 1–32, 55–60