Midterm III

Math 182, Section 1

Sample Problems

Question 1

List the first six terms of the sequence defined by

$$a_n = \frac{n}{2n+1}.$$

Does the sequence appear to have a limit? If so, find it.

Question 2

Determine whether the sequence

$$a_n = \frac{n^2-1}{n^2+1}$$

converges or diverges. If it converges, find the limit.

Question 3

Determine whether the sequence

$$\{\sqrt{n+2} - \sqrt{n}\}_{n=1}^{\infty}$$

converges or diverges. If it converges, find the limit.

Question 4

Determine whether the sequence

$$a_n = n2^{-n}$$

converges or diverges. If it converges, find the limit.

Question 5

Is the sequence

$$a_n = \frac{1}{3n+5}$$

increasing, decreasing, or neither?

Question 6

The Fibonacci sequence is defined recursively by the relationship $f_{n+1} = f_n + f_{n-1}$ and the starting conditions $f_1 = 1$, $f_2 = 2$. Now consider the ratios of adjacent elements in the Fibonacci sequence

$$a_n = \frac{f_{n+1}}{f_n}.$$

Show that $a_{n-1} = 1 + 1/a_{n-2}$ and, assuming that this sequence is in fact convergent, find the limit.

Question 7

Determine whether the series

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}$$

converges or diverges.

Question 8

Approximate the value of

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!}$$

to within 4 decimal places.

Question 9

For which values of p is the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$$

convergent?

Question 10

Show that

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

converges for all x, and deduce that

$$\lim_{n\to\infty} \frac{x^n}{n!} = 0$$

for all x.

Question 11

Determine whether

$$\sum_{n=1}^{\infty} ne^{-n^2}$$

is convergent or divergent.

Question 12

Determine whether

$$\sum_{n=1}^{\infty} \frac{1}{n^3 + n^2}$$

is convergent or divergent.

Question 13

Estimate the value of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

by taking the 10th partial sum. Give upper and lower bounds for the error and so give a more accurate estimate of the series.

Question 14

It is important to distinguish between the series

$$\sum_{n=1}^{\infty} n^b \qquad \text{and} \qquad \sum_{n=1}^{\infty} b^n.$$

What is the name of each of these series, and for what values of b does each converge?

Question 15

The meaning of the decimal representation of a number $0.d_1d_2d_3d_4\ldots$ where d_i is the ith digit of the number, and so is one of the integers 0 through 9, is that

$$0.d_1d_2d_3d_4\cdots = \frac{d_1}{10} + \frac{d_2}{10^2} + \frac{d_2}{10^2} + \ldots$$

Show that this series always converges, no matter what the digits are.

Question 16

Determine whether the series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$$

is convergent or divergent. If it is convergent, find its sum.

Question 17

Determine whether the series

$$\sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n}$$

is convergent or divergent. If it is convergent, find its sum.

Question 18

What is the value of c if

$$\sum_{n=2}^{\infty} (1+c)^{-n} = 2$$

Question 19

The *Sierpinski carpet* (see p713) is constructed by taking a square of side length one, and removing the open square of side length 1/3 from the middle. Of the remaining 8 squares of side length 1/3 we remove the open squares of side length 1/9 from the middle of each, and then the middle squares of side length 1/27 from the 64 remaining squares, and so on. Show that the total area of the removed squares is 1, which implies that the Sierpinski carpet has 0 area.

Question 20

Eliminate the parameter to find a Cartesian equation for the curve given by the parametric equations

$$x = \ln t, y = \sqrt{t}$$

for $t \geq 1$. Sketch the curve and indicate the direction the curve is traversed as t increases.

Question 21

Find the equation of the tangent to the curve given by the parametric equations

$$x = 2t + 1, y = \frac{1}{3}t^3 - t$$

at the pointe where $t = 3$.

Question 22

Find the length of the curve

$$y = \frac{x^4}{4} + \frac{1}{8x^2}$$

where $1 \leq x \leq 3$.

Question 23

A cow is tied to a silo of radius r by a rope just long enough to reach the opposite side of the silo. Find the area available for grazing by the cow. (See p654, questions 41 and 42.)

Question 24

Graph the curve given by the parametric equations

$$x = e^t \cos t, y = e^t \sin t$$

over the interval $0 \leq t \leq \pi$, and find the length of the curve.

Question 25

Find the area of the surface obtained by rotating the curve

$$x = t^3, y = t^2$$

where $0 \leq t \leq 1$ about the x-axis.

Question 26

Show that the polar curve $r = 4 + 2\sec\theta$ (called a *conchoid*) has the line $x = 2$ as a vertical asymptote. Use this fact to help sketch the conchoid.

Question 27

Find the slope of the tangent line to $r = \cos\theta + \sin\theta$ at $\theta = \pi/4$.

Question 28

Sketch the curve $r = 3(1 + \cos\theta)$ and find the area it encloses.

Question 29

Find a Cartesian equation which has the same graph as the polar equation $r\cos\theta = 1$.

Question 30

Find the length of the polar curve $r = 5\cos\theta$ over the interval $0 \leq \theta \leq 3\pi/4$.

Other Questions

Try:

Chapter 10 review, pp 688–689, exercises 1–38.
Chapter 11 review, pp 776–7, exercises 1–37.