Midterm II

Math 182, Section 1

24th October, 2001

Name:

Student ID Number:

Calculators are permitted unless they have a built-in algebra system.
You are permitted one two-sided letter-sized sheet of handwritten notes.

Part I - Short Answer

Write your final answer in the space provided. Partial credit will be given for correct working in incorrect answers.

Question 1

(a) Find

$$\int_0^2 \frac{1}{\sqrt{x}}\, dx.$$

(b) Does the integral

$$\int_0^2 \frac{\sin \frac{1}{x}}{\sqrt{x}}\, dx$$

converge or diverge?

(10 points)

Question 2

Find the centroid of the region between the curves

$$y = x^2 \qquad \text{and} \qquad y = x^4 - 2x^2$$

(10 points)

Question 3

Find the volume of solid whose base is the region between the curves

$$y = x^2 \qquad \text{and} \qquad y = x^4 - 2x^2$$

and whose cross-sections perpendicular to the x-axis are squares.

(10 points)

Question 4

Find the length of the curve

$$y = x^{3/2}$$

over the interval $0 \leq x \leq 4$.

(10 points)

Question 5

A spherical water tower has a reservoir with a 5m radius. If the bottom of the tower is 10m above ground level, how much work is required to pump it half full of water? (Assume water has a density of 1000kg/m^3 and acceleration due to gravity is 9.8m/s)

(10 points)

Part II - Long Answer

You must show all relevant working clearly and legibly. You will get no credit for a correct answer if there is no working, or if your working cannot be followed.

Question 6

Approximate

$$\int_0^1 e^{-x^2}\, dx$$

using the Trapezoid rule and Simpson's Rule with 4 subintervals.

Estimate the error E_T in the Trapezoid rule approximation.

How many subintervals would be required to approximate the interval to within 0.000001 of the actual value using the Trapezoid rule?

(25 points)

Question 7

Newton's Universal Law of Gravitation states that two masses m_1 and m_2 which are a distance r apart attract each other with a force

$$F = G\frac{m_1 m_2}{r^2},$$

where $G = 6.67 \times 10^{-11}$ N $\cdot$ m^2/kg is the gravitational constant.

Find the work required to launch a satellite which has a mass of 1000 kg to a height of 10^6 m above the surface of the Earth. Assume that the radius of the Earth is 6.37×10^6 m and the mass of the Earth is 5.98×10^{24} kg.

To escape Earth's gravitational pull completely, enough work must be done to move the satellite to a radius of $+\infty$. Find the amount of work required for the satellite in the previous part to escape Earth's gravitational pull.

(25 points)

Extra Credit - Long Answer

You must show all relevant working clearly and legibly. You will get no credit for a correct answer if there is no working, or if your working cannot be followed.

Question 7

A trough with semicircular cross-section has a diameter of 2 m and is 5m long. It is completely filled with water.

Find the hydrostatic force against the semicircular end. (Assume that the density of water is 1000 kg/m^3 and that acceleration due to gravity is 9.8 m/s)

(continued over ...)

Show that the area of a thin strip of the side wall at a depth x below the surface is approximately

$$A(x) = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, l \Delta x$$

where $l = 5$ m is the length of the trough, Δx is the veritcal extent of the thin strip, and $y = \sqrt{1 - x^2}$.

Conclude that the hydrostatic force on one side of the trough is given by

$$F = \int_0^1 \rho g x l \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx,$$

and calculate the force.

(20 points)