

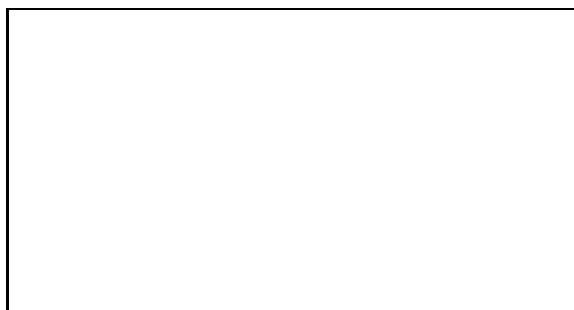
Sample Questions

Limits

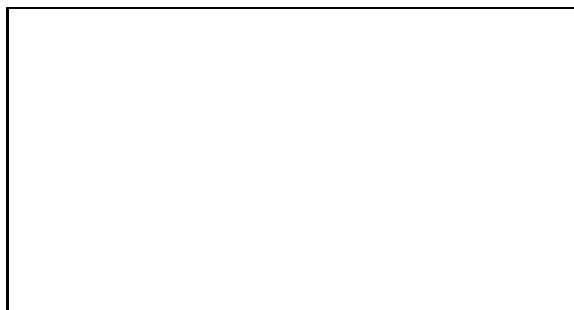
Question

In the space provided, sketch a graph which illustrates each of the following general situations:

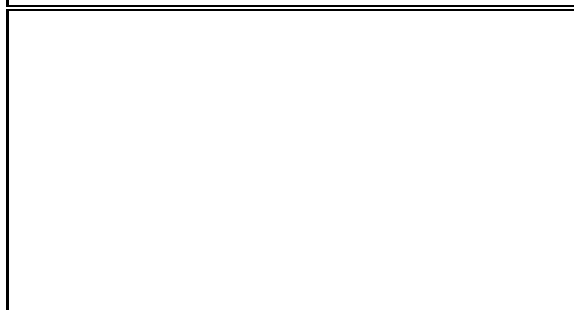
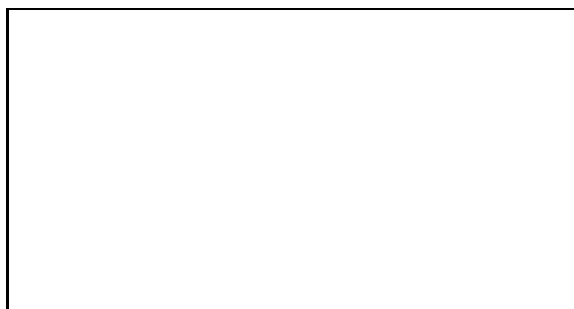
(a) $\lim_{x \rightarrow c} f(x) \neq f(c)$



(b) $\lim_{x \rightarrow c} f(x) = \infty$



(c) $\lim_{x \rightarrow c^-} f(x) \neq \lim_{x \rightarrow c^+} f(x)$

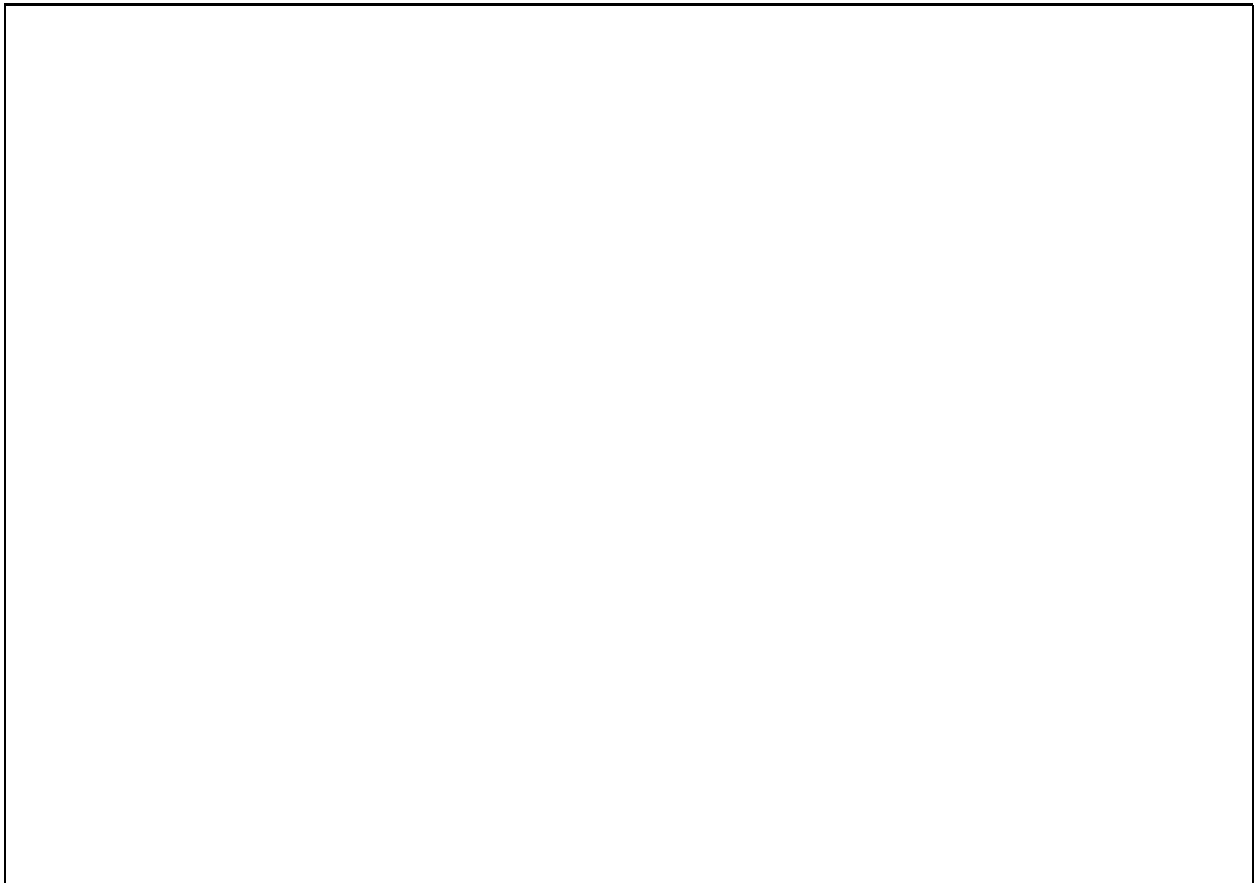


Question

Sketch the graph of a function $f(x)$ which satisfies the following conditions:

- a. $\lim_{x \rightarrow 1} f(x) = 1$
- b. $f(1) = 2$
- c. $\lim_{x \rightarrow 2^-} f(x)$ does not exist
- d. $f(x)$ is continuous on the interval $[-2, 1)$

Draw the graph clearly so that each of the above conditions is clearly illustrated. You do *not* need to provide a formula for the function.



Question

Write down the following limits:

(a) $\lim_{x \rightarrow 3} 5x^2 - 3 \tan(\pi x)$

(b) $\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta}$

(c) $\lim_{t \rightarrow 0} t \sin(1/t)$

Question

Write down the following limits:

(a) $\lim_{x \rightarrow 3} 4x^2 - 3 \sin(\pi x)$

(b) $\lim_{t \rightarrow 0} \frac{t}{\sin t}$

(c) $\lim_{h \rightarrow 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$

Question

Write down the following limits:

(a) $\lim_{x \rightarrow 1} \frac{x}{x+1} + \cos(\pi x)$

(b) $\lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$

(c) $\lim_{x \rightarrow \infty} \frac{\sqrt{x} - 1}{\sqrt{x} + x + 3}$

Question

Write down the following limits:

(a) $\lim_{x \rightarrow 3} \frac{4x^2}{x+1} - \cos(\pi x)$

(b) $\lim_{t \rightarrow \infty} \frac{3t^2 - 5t + 3}{7t^2 + 5t - 1}$

(c) $\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$

Question

Write down the following limits:

(a) $\lim_{x \rightarrow 3} \frac{x^2 - 1}{3x + 1} - 7 \cos(\pi x)$

(b) $\lim_{\theta \rightarrow 0} \frac{\theta}{\sin \theta}$

(c) $\lim_{t \rightarrow \infty} \frac{5t^{3/2} - 3t^2 + 1}{1 - 6t^{3/2}}$

Question

Do the following limits exist? If so, write down the value:

(a) $\lim_{x \rightarrow 1^+} \frac{|x-1|}{x-1}$

(b) $\lim_{x \rightarrow 1^-} \frac{|x-1|}{x-1}$

(c) $\lim_{x \rightarrow 1} \frac{|x-1|}{x-1}$

Question

Consider the function

$$f(x) = \begin{cases} \cos x & x \geq 0 \\ ax + b & x < 0 \end{cases}$$

For which values of b is $f(x)$ continuous at $x = 0$?

Question

Consider the function

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & x \neq 0 \\ 12 & x = 0. \end{cases}$$

The limit

$$\lim_{x \rightarrow 0} f(x)$$

- Does not exist, because $f(x)$ oscillates wildly.
- Does not exist, because although the left-hand and right-hand limits exist, they are not equal.
- Does exist and is equal to 0, because the Sandwich Theorem applies with $-x^2 \leq f(x) \leq x^2$.
- Does exist and is equal to 12, because $f(0) = 12$.
- None of the above.

Question

Consider the function

$$f(x) = \cos x - x$$

on the interval $[0, \pi]$.

Which of the following statements are false, and which are true:

- $f(x)$ is continuous.
- $f(x) = 0$ at some point in $[0, \pi]$.

Question

On which interval must the function $f(x) = 2 + x^2 - x^3$ have a root? (Use the Intermediate Value Theorem)

- a. $[-1, 0]$
- b. $[0, 1]$
- c. $[1, 2]$
- d. $[2, 3]$
- e. None of the above.