Final

Math 181, Section 3

May 8th, 2000

Name:

Calculators are permitted unless they have a built-in algebra system.
You are permitted two two-sided, letter-sized sheets of handwritten notes.

At least one mark will be taken off for every constant of integration that is missing!

Part I - Short Answer

Write your answer in the space provided. No partial credit.

Question 1

Write down the following derivatives:

(a) $\frac{d}{dx}(\sin(x^2+1))$

(b) $\frac{d}{dt}(6t^2 - 4t + \sqrt[3]{t} - \frac{1}{t^2})$

(c) $\frac{d}{dx}((x^3+1)^2(5x^2-2)^{100})$

(9 points)

Question 2

Find the value of the following integrals:

(a) $\int 2x\cos(x^2+1)\ dx$

(b) $\int_0^1 6t^2 - 4t + \sqrt[3]{t} - \frac{1}{t^2}\ dx$

(c) $\int_0^{\pi} \cos^2\theta\ d\theta$

(9 points)

Question 3

Write down the following limits:

(a) $\lim_{x \to 0} \dfrac{3x^2 + 5}{7x - 1} + \cos 2x$

(b) $\lim_{t \to 0^+} \dfrac{1}{t}$

(c) $\lim_{h \to 0} \dfrac{\left(\frac{1}{3+h} - \frac{1}{3}\right)}{h}$

(b) $\lim_{x \to -\infty} \dfrac{7x^2 - 3}{5x^2 - 2x + 1}$

(8 points)

Question 4

If

$$xy^3 - 3y^2 + 5x^3 \sin y = 0$$

find $\frac{dy}{dx}$.

(5 points)

Question 5

A weight hanging from a spring is pulled 0.1 metres downwards from its equillibrium point. If the force exerted by the spring is

$$F(x) = 5x$$

Newtons, where x is the distance of the weight from equillibrium, find the work done in moving the spring.

(5 points)

Question 6

Sketch the graph of a function $f(x)$ which has all of the following properties:

a. $f(x)$ is continuous.

b. $f(x)$ is increasing on $(-\infty, 0]$ and $[2, \infty)$.

c. $f(x)$ is decreasing on $[0, 2]$.

d. $f(x)$ is concave up on $(-\infty, -1]$ and $[0, 2]$.

e. $f(x)$ is concave down on $[-1, 0]$ and $[2, \infty)$.

(5 points)

Question 7

Consider the function

$$f(x) = \cos x - x$$

on the interval $[0, \pi]$.

Which of the following statements are false, and which are true:

a. $f(x)$ is continuous.

b. $f(x) = 0$ at some point in $[0, \pi]$.

c. $\int f(x)\ dx = \sin x - x^2/2$.

d. $f'(x) = -\frac{\pi+2}{\pi}$ at some point in $[0, \pi]$.

e. $f'(x) = 0$ at some point in $[0, \pi]$.

(5 points)

Question 8

Write down the integral which corresponds to the Riemann sum

$$\lim_{\|P\|\to 0} \sum_{k=1}^{n} 2c_k \sin c_k \Delta x_k$$

taken over the interval $[1, 2]$. (Don't try to calculate its value.)

(4 points)

Part II - Long Answer

You must show all relevant working. You will get no credit for a correct answer if there is no working.

Question 9

A thin plate is bounded by the curves $y = x^2$, $y = \frac{1}{x^2}$ and $x = 2$. The density of the plate at is given by the function $\delta(x) = \frac{1}{x}$.

(a) Sketch the region the plate covers in the xy-plane.

(b) Find the centre of mass of the plate.

(15 points)

Question 10

A particle moves from right to left along a parabola $y = \sqrt{-x}$ in such a way that its x-coordinate (in metres) decreases at a rate of 8ms^{-1}.

How fast is the angle θ between the line joining the particle to the origin and the x-axis changing when $x = -4$?

(15 points)

Question 11

A rectangular sheet of paper is rolled into a cylinder with open ends. If the perimeter of the sheet of paper is 36 centimeters, what side lengths give a cylinder with the greatest volume?

(10 points)

Question 12

Verfiy, using the limit definition of the derivative, that for any functions $f(x)$ and $g(x)$,

$$\frac{d}{dx}(f(x) - g(x)) = \frac{d}{dx}f(x) - \frac{d}{dx}g(x).$$

(10 points)

Extra Credit

You must show all relevant working. If you will get no credit for a correct answer if there is no working.

Question 13

The product rule says that

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

(a) Integrate both sides and re-arrange to show that:

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

(Hint: uv is an antiderivative of $\frac{d}{dx}(uv)$.)

(b) Let $u = x$ and $\frac{dv}{dx} = \sin x$. What are possible values of $\frac{du}{dx}$ and v?

(c) Use part (b) to show that

$$\int x \sin x \; dx = -x \cos x + \sin x + C.$$

(d) Verify (c) by differentiation.

(20 points)