**Question 1**

(a) $\frac{d}{dx}(\cos(3x^2+1)) = -\sin(3x^2+1)\frac{d}{dx}(3x^2+1) = -6x\sin(3x^2+1)$
(b) $\frac{d}{dx}(5\cos^2 t) = 5(2\cos t)\frac{d}{dx}(\cos t) = -10\cos t \sin t$
(c) $\frac{d}{dx}(\sqrt{1-x^2}) = \frac{1}{2}(1-x^2)^{1/2}\frac{d}{dx}(1-x^2) = \frac{-x}{\sqrt{1-x^2}}$
*These are all straightforward chain rule examples.*

**Question 2**

(a) 1/4
(b) $-\infty$
(c) 0.
*See section 3.5 if you had difficulty with these.*

**Question 3**

$$h'(x) = f'(g(x))g'(x)$$

so

$$h'(1) = f'(g(1))g'(1) = f'(5)\cdot 1 = -1$$

*This is the chain rule again.*

**Question 4**

C

A critical point has either $f'(c) = 0$ or $f'(c)$ does not exist; so if $f$ is differentiable at $a = c$, $f'(c)$ exists, and so it must be 0 if it is critical.

*Not every critical point is a maxima or minima; and $f'(c)$ can be 0 at a critical point. See section 3.1.*

**Question 5**

The hard-but-straightforward way:

$$\begin{aligned}
\frac{d}{dx}(\sqrt{xy}) &= \frac{d}{dx}(5x^2-3y^2) \\
\frac{1}{2}(xy)^{-1/2}\frac{d}{dx}(xy) &= 10x - 6y\frac{dy}{dx} \\
\frac{1}{2\sqrt{xy}}\left(x\frac{dy}{dx} - y\frac{d}{dx}x\right) &= 10x - 6y\frac{dy}{dx} \\
(x + 12y\sqrt{xy})\frac{dy}{dx} &= 20x\sqrt{xy} + y \\
\frac{dy}{dx} &= \frac{20x\sqrt{xy}+y}{x+12y\sqrt{xy}}
\end{aligned}$$

The slightly easier, but sneakier way:

$$xy = (5x^2-3y^2)^2 = 25x^4 - 30x^2y^2 + 9y^4$$

so

$$\begin{aligned}
\frac{d}{dx}(xy) &= \frac{d}{dx}(25x^4 - 30x^2y^2 + 9y^4) \\
x\frac{dy}{dx} + y &= 100x^3 - 30x^2\frac{d}{dx}(y^2) - y^2\frac{d}{dx}(30x^2) + 36y^3\frac{dy}{dx} \\
x\frac{dy}{dx} + y &= 100x^3 - 60x^2y\frac{dy}{dx} - 60xy^2 + 36y^3\frac{dy}{dx} \\
(x + 60x^2y - 36y^3)\frac{dy}{dx} &= 100x^3 - 60xy^2 - y \\
\frac{dy}{dx} &= \frac{100x^3 - 60xy^2 - y}{x + 60x^2y - 36y^3}
\end{aligned}$$

*Implicit Differentiation, see section 2.7.*

**Question 6**

Still can't import graphs into these documents ... *sigh*

**Question 7**

$$\begin{aligned}
f'(x) &= \frac{2}{3}\left(\frac{2x}{x^2+1}\right)^{-1/3}\frac{d}{dx}\left(\frac{2x}{x^2+1}\right) \\
&= \frac{2}{3}\left(\frac{x^2+1}{2x}\right)^{1/3}\frac{(x^2+1)2 - 2x(2x)}{(x^2+1)^2} \\
&= \frac{2}{3}\left(\frac{x^2+1}{2x}\right)^{1/3}\frac{2 - 2x^2}{(x^2+1)^2} \\
&= \frac{4(1-x^2)}{6x(x^2+1)^{5/3}}
\end{aligned}$$

The derivative is 0 when the numerator is 0, ie. $1 - x^2 = 0$, ie. when $x = \pm 1$.
The definative is undefined when the demonimator is 0, ie. when $x(x^2+1)^{5/3} = 0$, ie. when $x = 0$.
So the critical points are at $x = -1$, 0 and 1. The $y$-values are 1, 0 and 1 respectively.
So the critical points are $(-1, 1)$, $(0, 0)$ and $(1, 1)$.

$$\begin{aligned}
\lim_{x\to\infty}\left(\frac{2x}{x^2+1}\right)^{2/3} &= \left(\lim_{x\to\infty}\frac{2x}{x^2+1}\right)^{2/3} \\
&= \left(\lim_{x\to\infty}\frac{2/x}{1+1/x^2}\right)^{2/3} \\
&= 0^{2/3} = 0
\end{aligned}$$

Similarly

$$\lim_{x\to-\infty} f(x) = 0.$$

Putting the above three sections together, the absolute maximum value is clearly 1; while the absolute minimum is 0 (the asymptotes at infinity do not affect this, since they are not *less than* 0).

*This puts together calculations from sections 3.1 and 3.5. Many calculators do not graph this function correctly.*

**Question 8**

We are told that $\frac{d\theta}{dt} = 0.5$. And form the diagram and the definitions of trig functions $\tan\theta = x/1 = x$. We want to know $\frac{dx}{dt}$. Differentiating implicitly:

$$\frac{dx}{dt} = \frac{d}{dt}\tan\theta = \sec^2\theta\frac{d\theta}{dt}$$

In the first part, $x = 0$ and so $\theta = 0$, so

$$\frac{dx}{dt} = (\sec^2 0)(0.5) = 0.5\text{km/s}.$$

In the second part, $x = 1$, so $\theta = \pi/4$, so

$$\frac{dx}{dt} = (\sec^2 \pi/4)(0.5) = 1\text{km/s}.$$

*This is a related relates question, see section 2.7.*

**Question 9**

The formula for the cost of the cable is

$$c = 10000(5 - x) + 20000y = 10000(5 - x + 2\sqrt{x^2 + 1}).$$

We want to minimize this. Clearly from the diagram, $0 \leq x \leq 5$, so these are our constraints.
No finding the critical points:

$$\frac{dc}{dx} = 10000(-1 + 2\frac{1}{2}(x^2+1)^{-1/2}\frac{d}{dx}(x^2+1)) = 10000(-1 + \frac{2x}{\sqrt{x^2+1}})$$

which is always defined and is 0 when:

$$\begin{aligned}
-1 + \frac{2x}{\sqrt{x^2+1}} &= 0 \\
-\sqrt{x^2+1} - 2x &= 0 \\
2x^2 &= \sqrt{x^2+1} \\
4x^2 &= x^2 + 1 \\
x^2 &= 1/3 \\
x &= \pm\frac{1}{\sqrt{3}}
\end{aligned}$$

We throw out $-1/\sqrt{3}$ as it is not within the constraints. So we need to check $x = 0$, $1/\sqrt{3}$ and 5.

$$\begin{aligned}
c(0) &= 10000(5 - 0 + 2\sqrt{1+0}) = 70000 \\
c(1/\sqrt{3}) &= 10000(5 - 1/\sqrt{3} + 2\sqrt{1 + 1/3}) = 67320 \\
c(5) &= 10000(5 - 5 + 2\sqrt{1+25}) = 20000\sqrt{26} = 101980
\end{aligned}$$

So the minimum cost is \$67320.
*This is an optimization problem, see section 3.6.*

## Question 10

Using implicit differentiation,

$$\begin{aligned}\frac{d}{dx}(xy) &= \frac{d}{dx}(c)\\ x\frac{dy}{dx} + y &= 0\\ \frac{dy}{dx} &= \frac{-y}{x}\end{aligned}$$

Using implicit differentiation,

$$\begin{aligned}\frac{d}{dx}(x^2 - y^2) &= \frac{d}{dx}(d)\\ 2x - 2y\frac{dy}{dx} &= 0\\ \frac{dy}{dx} &= \frac{2x}{2y} = \frac{x}{y}\end{aligned}$$

So if two of these curves intersect at $(x_0, y_0)$, the slope of the tangent lines are

$$m_1 = \frac{-y_0}{x_0}$$

and

$$m_2 = \frac{x_0}{y_0}$$

respectively, and

$$m_1 = \frac{-1}{m_2}$$

so the tangent lines are perpendicular.