

# Midterm I

Math 181, Section 3  
February 18th, 2000

Name:

Student ID:

Calculators are permitted unless they have a built-in algebra system.  
You are permitted one two-sided letter-sized sheet of handwritten notes.

## Part I - Short Answer

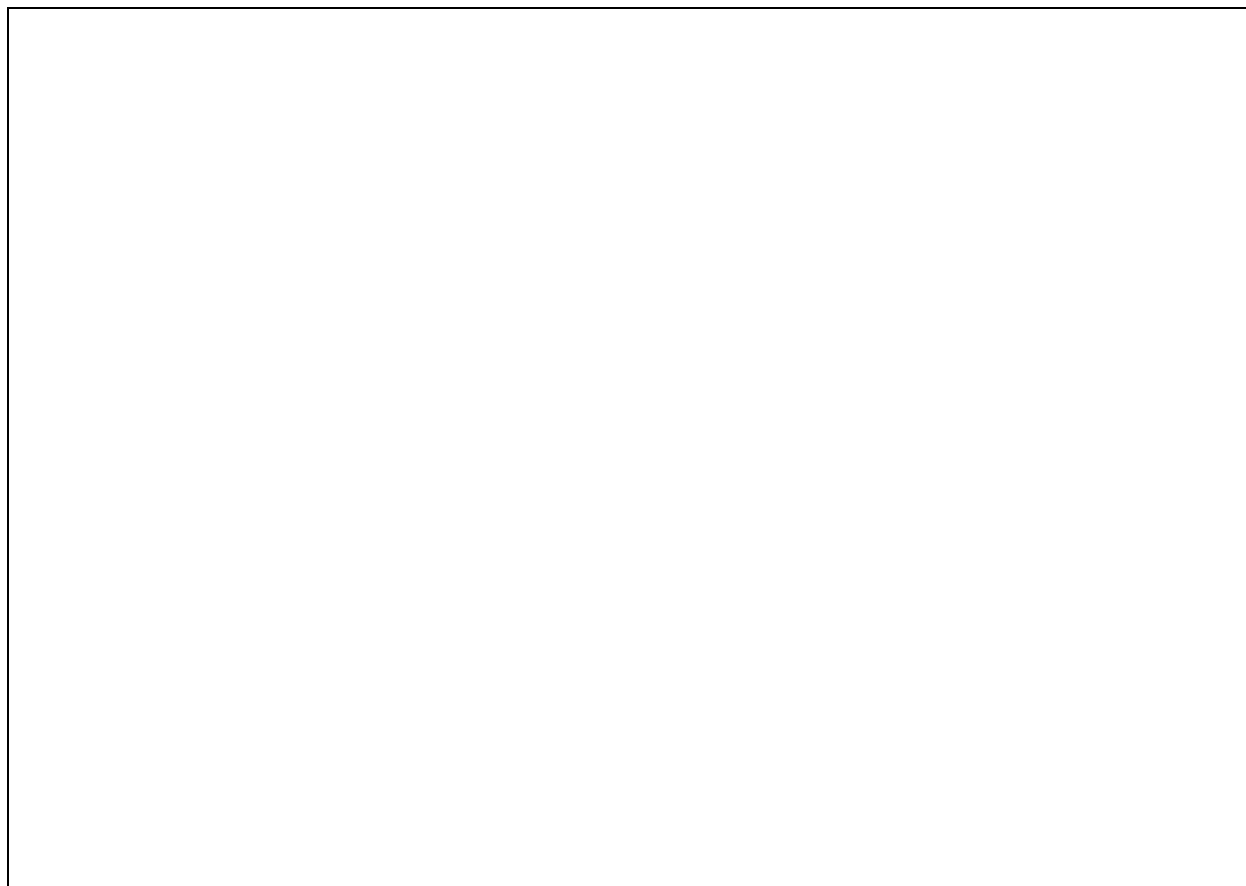
Write your answer in the space provided. No partial credit.

### Question 1

Sketch the graph of a function  $f(x)$  which satisfies the following conditions:

- $\lim_{x \rightarrow 1} f(x) = 1$
- $f(1) = 2$
- $\lim_{x \rightarrow 2^-} f(x)$  does not exist
- $f(x)$  is continuous on the interval  $[-2, 1)$
- $f(x)$  is not differentiable at  $x = -1$ .

Draw the graph clearly so that each of the above conditions is clearly illustrated. You do *not* need to provide a formula for the function.



(10 points)

**Question 2**

Write down the following limits:

(a)  $\lim_{x \rightarrow 3} 5x^2 - 3 \tan(\pi x)$

(b)  $\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta}$

(c)  $\lim_{t \rightarrow 0} t \sin(1/t)$

(9 points)

**Question 3**

Write down the following derivatives:

(a)  $\frac{d}{dx}(16x^4 + 12x^2 - 3x^{-1})$

(b)  $\frac{d}{d\theta}(\tan \theta \csc \theta)$

(c)  $\frac{d}{dt}((2t^3 + 1)(7 - 3t))$

(12 points)

**Question 4**

Do the following limits exist? If so, write down the value:

(a)  $\lim_{x \rightarrow 1^+} \frac{|x-1|}{x-1}$

(b)  $\lim_{x \rightarrow 1^-} \frac{|x-1|}{x-1}$

(c)  $\lim_{x \rightarrow 1} \frac{|x-1|}{x-1}$

(9 points)

**Question 5**

If  $h(x) = f(x)/g(x)$ ,  $f(5) = 1$ ,  $f'(5) = -1$ ,  $g(5) = 3$  and  $g'(5) = 5$ , write down  $h'(5)$ .

(5 points)

**Question 6**

On which interval must the function  $f(x) = 2 + x^2 - x^3$  have a root? (Use the Intermediate Value Theorem)

- a.  $[-1, 0]$
- b.  $[0, 1]$
- c.  $[1, 2]$
- d.  $[2, 3]$
- e. None of the above.

(5 points)

## Part II - Long Answer

You must show all relevant working. You will get no credit for a correct answer if there is no working.

### Question 7

The upper half of a circle of radius 1 is given by the equation  $y = \sqrt{1 - x^2}$ . Using limit techniques, find the derivative.

Find the equation of a tangent through a general point  $(c, \sqrt{1 - c^2})$  on the semicircle.

Find the tangent line to the semicircle which passes through the point  $(\sqrt{2}, 0)$ .

(20 points)

**Question 8**

The position of an object is given by the formula

$$s(t) = \frac{4t}{t^2 + 1}.$$

Find the formula of the velocity of the object.

Find the velocity at  $t = 0$  and  $t = 1$ .

(15 points)

### Question 9

The product rule says that the first derivative of  $y = uv$  is given by

$$\frac{dy}{dx} = \frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}.$$

Use the product rule to find a formula for the *second* derivative of  $y$ .

Use this formula to find the second derivative of  $y = (x^2 + 1) \cos x$ .

(15 points)

## Extra Credit - Long Answer

You must show all relevant working. You will get no credit for a correct answer if there is no working.

### Question 10

This continues Question 9.

Find formulas for the third and fourth derivatives of  $y = uv$ . Try and collect terms to write in the simplest possible way.

There is a pattern in the coefficients which you should recognise. What is it?

Write down the general formula for the  $n$ th derivative of  $y = uv$ .

(15 points)