Midterm II

Math 181, Section 3

March 20th, 2000

Name:

Student ID:

Calculators are permitted unless thay have a built-in algebra system.
You are permitted one two-sided letter-sized sheet of handwritten notes.

Part I - Short Answer

Write your answer in the space provided. No partial credit.

Question 1

Write down the following derivatives

(a) $\frac{d}{dx}(\cos(3x^2+1))$

(b) $\frac{d}{dt}(5\cos^2 t)$

(c) $\frac{d}{dx}(\sqrt{1-x^2})$

(9 points)

Question 2

Write down the following limits:

(a) $\lim_{x\to\infty} \frac{3x^4-5x^2+7x}{12x^4-5x^3+x}$

(b) $\lim_{x\to-\infty} \frac{3x^4-5x^2+7x}{12x^3-5x^2+1}$

(c) $\lim_{x\to\infty} \frac{3x^4-5x^2+7x}{12x^5-5x^4+x^2}$

(9 points)

Question 3

If $h(x) = f(g(x))$, $f(1) = 1$, $f(5) = 2$, $f'(1) = 3$, $f'(5) = -1$, $g(1) = 5$ and $g'(1) = 1$, write down $h'(1)$.

(8 points)

Question 4

A function $f(x)$ has a critical point at $x = c$. Which of the following must be true:

(a) $f(x)$ has either an absolute maximum or absolute minimum at $x = c$.
(b) $f(x)$ has either a local maximum or a local minimum at $x = c$.
(c) If $f(x)$ is differentiable at $x = c$, then $f'(c) = 0$.
(d) $f'(c)$ does not exist.
(e) None of the above.

(6 points)

Question 5

If $\sqrt{xy} = 5x^2 - 3y^2$, write down $\frac{dy}{dx}$.

(10 points)

Question 6

Sketch a continuous function defined on the interval $(0, 1]$ which has no absolute maximum.

(8 points)

Part II - Long Answer

You must show all relevant working. You will get no credit for a correct answer if there is no working.

Question 7

Consider the function

$$f(x) = \left(\frac{2x}{x^2+1}\right)^{2/3}.$$

Differentiate $f(x)$.

Find the critical points of $f(x)$.

Find

$$\lim_{x\to\infty} f(x) \qquad \text{and} \qquad \lim_{x\to-\infty} f(x)$$

Find the absolute maxima and minima of $f(x)$, or explain why they do not exist.

(20 points)

Question 8

A lighthouse is 1 km off a straight shoreline, as illustrated, sweeping the shore with its beam of light. The ray of light revolves at a constant rate of 0.5 radians per second.

How fast is the point where the beam of light hits the shore moving when it is:
(a) At the point on the shore closest to the lighthouse.

(b) 1km from the closest point.

(15 points)

Question 9

A telephone cable is to be laid from an exchange at a coastal village to the lighthouse from the previous question. The village is 5 km from the point on the shore which is closest to the lighthouse. Running cable along the shore costs \$10,000 per km, while running cable underwater costs \$20,000 per km.

If the cable is run along the shore from the village, and then directly out to the lighthouse, as indicated, find an expression for the total cost of laying the cable.

Find the minimum cost of laying the cable.

(15 points)

Extra Credit - Long Answer

You must show all relevant working. You will get no credit for a correct answer if there is no working.

Question 10

In many physical and engineering applications we encounter two families of curves, such that when any two curves intersect, they intersect at right angles. Such families are called *orthogonal nets*.

For example, the lines of magnetic force and magnetic potential around a magnet form such a pair of families.

In this question, we will show that the two families of hyperbolas

$$xy = c \qquad \text{and} \qquad x^2 - y^2 = d$$

(where c and d are constant parameters) form an orthogonal net.

Find $\frac{dy}{dx}$ for a curve of the form

$$xy = c$$

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Find $\frac{dy}{dx}$ for a curve of the form

$x^2 - y^2 = d$

If a curve $xy = c$ and a curve $x^2 - y^2 = d$ intersect at a point (x_0, y_0), show that the tangent lines of each curve at that point are perpendicular.

(15 points)