# Midterm III

Math 181, Section 3

April 28th, 2000

Name:

Student ID:

Calculators are permitted unless thay have a built-in algebra system.
You are permitted one two-sided letter-sized sheet of handwritten notes.
You will lose at least one mark for each "$+C$" that you fail to put in where it is needed.

# Part I - Short Answer

Write your answer in the space provided. No partial credit.

### Question 1

Write down the following indefinite integrals

(a) $\int \cos^2 x \; dx$

(b) $\int 3t^2 + \sqrt{t} - \frac{1}{t} \; dt$

(c) $\int \frac{x}{\sqrt{x^2+1}} \; dx$

(9 points)

### Question 2

Write down the following definite Integrals

(a) $\int_0^{\pi/2} \cos^2 x \; dx$

(b) $\int_1^4 3t^2 + \sqrt{t} - \frac{1}{t} \; dt$

(c) $\int_0^{\pi/2} \sin\theta\sqrt{1+\cos\theta} \; d\theta$

(9 points)

**Question 3**

Write down the value of

$$\frac{d}{dt}\int_1^t \cos u^2 \, du$$

(8 points)

**Question 4**

Write down the average value of the function

$$f(x) = \frac{x}{\sqrt{x^2+1}}$$

on the interval $[0, 3]$.

(8 points)

**Question 5**

Which points $x = c$ satisfy the mean value theorem for the function

$$f(x) = x^3 - x$$

on the interval $[-2, 2]$.

(8 points)

**Question 6**

A rectangular swimming pool is 30 ft wide and 50 ft long. The table below shows the depth of the water $h(x)$ at 10 ft intervals from one end of the pool to the other.

| Position $x$ ft | 0 | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|
| Depth $h(x)$ ft | 6.0 | 9.1 | 10.5 | 11.5 | 12.3 | 13.0 |

(a) Estimate the volume of the pool using left-endpoint values.

(b) Estimate the volume of the pool using right-endpoint values.

(8 points)

# Part II - Long Answer

You must show all relevant working. You will get no credit for a correct answer if there is no working.

### Question 7

Solve the initial value problem

$$\frac{d^2y}{dx^2} = 4\sec^2 2x \tan 2x, \quad y'(0) = 4, y(0) = -1$$

(15 points)

**Question 8**

Consider the generating region bounded by $y = \sqrt{x}$, $x = 4$ and the $x$-axis.

(a) Find the volume of revolution obtained by rotating this region about the $x$-axis.

(b) Find the volume of revolution obtained by rotating this region about the $y$-axis.

(c) Find the volume of revolution obtained by rotating this region about the line $x = 4$.

(d) Find the volume of revolution obtained by rotating this region about the line $y = 2$.

(20 points)

**Question 9**

Consider the function

$$f(x) = \frac{x^2 - 49}{x^2 + 5x - 14}$$

Sketch the graph of this function, showing *all* relevant calculations. Clearly mark any and all critical points, asymptotes and inflection points of the graph.

(15 points)

# Extra Credit - Long Answer

You must show all relevant working. You will get no credit for a correct answer if there is no working.

### Question 10

At present we cannot evaluate the integral

$$\int \frac{1}{x}\, dx.$$

In this question we will investigate this integral. To do this we consider the function

$$F(x) = \int_1^x \frac{1}{t}\, dt$$

defined for $x > 0$.

(a) Show that $F(x)$ is an antiderivative of $\frac{1}{x}$, and that $F(1) = 0$.

(b) Show that $F(ax)$ is also an antiderivative of $\frac{1}{x}$.

(c) Part (b) imples that $F(ax) = F(x) + C$. Plug $x = 1$ into this equation and show that $F(ax) = F(x) + F(a)$.

(d) Show that $F(x^n) = nF(x)$. (Hint: differentiate $F(x^n)$ and $nF(x)$. What do you notice about the derivatives? Then proceed as in Part (c))

(e) Notice that these rules match the rules for logarithms:

$$\log_b ax = \log_b x + \log_b a \quad \log_b x^n = n \log_b x \quad \log_b 1 = 0$$

So we might guess that $F(x)$ is a logarithm of some sort. Let $e$ be the number so that $F(e) = 1$, show that

$$F(x) = \log_e x$$

(Hint: any $x > 0$ can be written as $x = e^{\log_e x}$.)

(25 points)