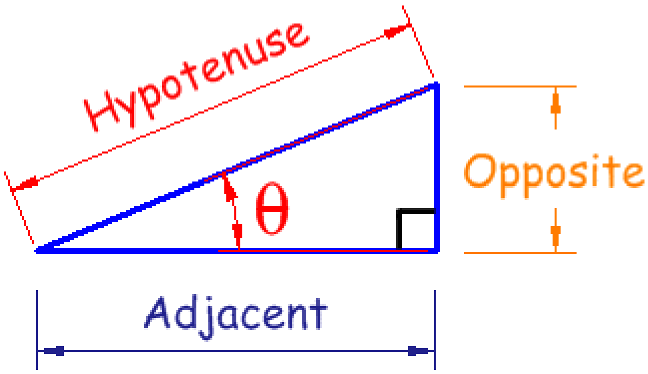
5.2 Right Triangle Trigonometry

This is a very important section since we are giving definitions for the six trigonometric functions you be using throughout the rest of this course and beyond. We need to first start with a drawing of a right triangle. The following definitions only apply to RIGHT TRIANGLES.

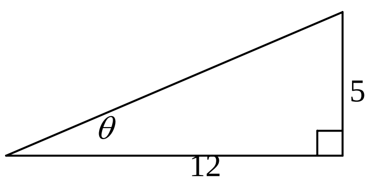


$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \csc\theta = \frac{hypotenuse}{opposite}$$

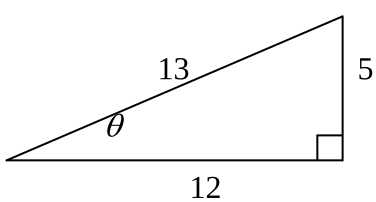
$$\cos\theta = \frac{adjacent}{hypotenuse} \qquad \sec\theta = \frac{hypotenuse}{adjacent}$$

$$\tan\theta = \frac{opposite}{adjacent} \qquad \cot\theta = \frac{adjacent}{opposite}$$

EXAMPLE: Find the exact value of the 6 trig functions using the following figure:



First we need to find the missing side. In this case it is the hypotenuse. In order to find this we need to use the formula $a^2 + b^2 = c^2$. The side opposite the right angle is always side c. So we have $12^2 + 5^2 = c^2$. Simplifying we will get $144 + 25 = c^2$, or $169 = c^2$. We will get $c = \pm 13$, however our answer is c = 13 since we can't have a negative side. Now we are ready to write our 6 trig functions. Here the hypotenuse is 13, opposite is 5, and the adjacent side is 12. We will put these into the formulas and that will be our answers.

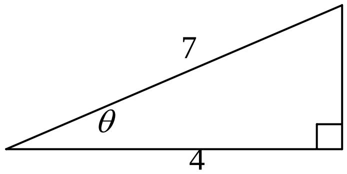


$$\sin\theta = \frac{5}{13} \qquad \csc\theta = \frac{13}{5}$$

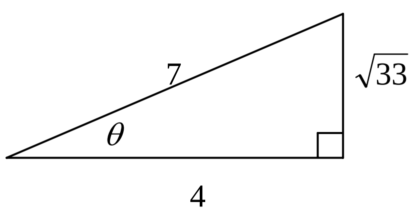
$$\cos\theta = \frac{12}{13} \qquad \sec\theta = \frac{13}{12}$$

$$\tan\theta = \frac{5}{12} \qquad \cot\theta = \frac{12}{5}$$

EXAMPLE: Find the exact value of the 6 trig functions using the following figure:



First we need to find the missing side. In this case it is the hypotenuse. In order to find this we need to use the formula $a^2 + b^2 = c^2$. The side opposite the right angle is always side c, which is 7 in our figure. So we have $4^2 + b^2 = 7^2$. Simplifying we will get $16 + b^2 = 49$, or $33 = c^2$. We will get $c = \sqrt{33}$. Now we are ready to write our 6 trig functions. Here the hypotenuse is 7, opposite is $\sqrt{33}$, and the adjacent side is 4. We will put these into the formulas and that will be our answers. The answer for cosecant is $\frac{7}{\sqrt{33}}$. This needs to be rationalized by multiplying top and bottom by the square root of 33: $\frac{7}{\sqrt{33}} \cdot \frac{\sqrt{33}}{\sqrt{33}} = \frac{7\sqrt{33}}{33}$. You want to always rationalize so that there is no square root in the denominator.



$\sin\theta = \frac{\sqrt{33}}{7}$ $\qquad$ $\csc\theta = \frac{7}{\sqrt{33}} = \frac{7\sqrt{33}}{33}$

$\cos\theta = \frac{4}{7}$ $\qquad$ $\sec\theta = \frac{7}{4}$

$\tan\theta = \frac{\sqrt{33}}{4}$ $\qquad$ $\cot\theta = \frac{4}{\sqrt{33}} = \frac{4\sqrt{33}}{33}$

Basic Trigonometric Identities

$\csc\theta = \frac{1}{\sin\theta}$ $\qquad$ $\sec\theta = \frac{1}{\cos\theta}$ $\qquad$ $\cot\theta = \frac{1}{\tan\theta}$

We can use the above definitions to enter the following into the calculator:

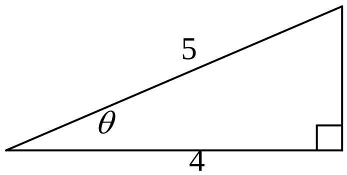
EXAMPLE: Use a calculator to evaluate the 6 trigonometric functions with $\theta = 50^\circ$. Round to 3 places.

Before we can do this we must make sure our calculator is in DEGREE mode. If you have a graphing calculator, go into the MODE and move your cursor over degrees and hit enter to highlight it. If you have a calculator like the TI-30xa, there is a DRG key towards the top. You will need to hit the second key in order to get it. You want to see a DEG at the top of the screen. To find cosecant, secant, or cotangent you will need to use the trig identities defined above. For example: $\csc 50 = \frac{1}{\sin 50} = \frac{1}{0.766} = 1.305$.

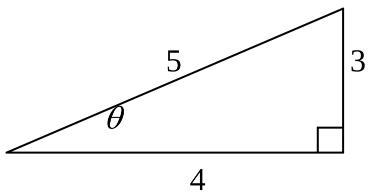
$\sin 50 = 0.766$ $\qquad$ $\cos 50 = 0.643$ $\qquad$ $\tan 50 = 1.192$ $\qquad$ $\csc 50 = 1.305$ $\qquad$ $\sec 50 = 1.556$ $\qquad$ $\cot 50 = 0.839$

EXAMPLE: Given $\cos\theta = \frac{4}{5}$, find the other 5 trigonometric values.

For a problem like this let's first draw a triangle and label the adjacent and hypotenuse since we know that cosine is equal to the adjacent divided by the hypotenuse:



From using $a^2 + b^2 = c^2$ we will find that the missing side is equal to 3. Now we can label that and find our trig values:



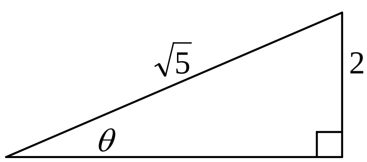
$\sin\theta = \frac{3}{5}$ $\csc\theta = \frac{5}{3}$

$\cos\theta = \frac{4}{5}$ $\sec\theta = \frac{5}{4}$

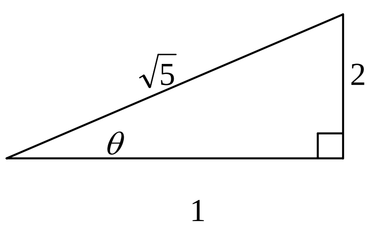
$\tan\theta = \frac{3}{4}$ $\cot\theta = \frac{4}{3}$

EXAMPLE: Given $\csc\theta = \frac{\sqrt{5}}{2}$, find the other 5 trigonometric values.

The picture of this looks like:



From using $a^2 + b^2 = c^2$ we will find that the missing side is equal to 1. The other values are:



$\sin\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\csc\theta = \frac{\sqrt{5}}{2}$

$\cos\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\sec\theta = \sqrt{5}$

$\tan\theta = \frac{2}{1} = 2$ $\cot\theta = \frac{1}{2}$

Other Trigonometric Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ $\sin^2\theta + \cos^2\theta = 1$

EXAMPLE: Find the exact value without a calculator: $\tan 70 - \dfrac{\sin 70}{\cos 70}$.

Using the identities we can change the second term into $\tan 70$. Then we will have: $\tan 70 - \tan 70 = 0$. So the answer to the whole problem is zero.

EXAMPLE: Find the exact value without a calculator: $\cos^2 40 + \dfrac{1}{\csc^2 40}$.

The second term is the same as $\sin^2 40$. So you will have $\cos^2 40 + \sin^2 40$ which is just 1.

Complementary Angle Theorem

$\sin\theta = \cos(90-\theta)$ $\csc\theta = \sec(90-\theta)$

$\cos\theta = \sin(90-\theta)$ $\sec\theta = \csc(90-\theta)$

$\tan\theta = \cot(90-\theta)$ $\cot\theta = \tan(90-\theta)$

EXAMPLE: Write the following as an equivalent cosine expression: $\sin 33$.

We can use the Complementary Angle Theorem for this. We will use $\sin\theta = \cos(90-\theta)$. Here $\theta = 33$. Substituting we get $\sin 33 = \cos(90-33)$. So our answer is: $\sin 33 = \cos 57$. If you put both of these in your calculator you should get the same decimal.

EXAMPLE: Simplify and find the EXACT value: $\tan 35 \cdot \sec 55 \cdot \cos 35$

We need to use the Complementary Angle Theorem to get rid of the 55 angle. The only formula we can use for secant is $\sec\theta = \csc(90-\theta)$. We will get: $\sec 55 = \csc(90-55)$. We will get: $\sec 55 = \csc 35$. Now our problem becomes: $\tan 35 \cdot \csc 35 \cdot \cos 35$. Let's now change everything into sines and cosines:

$\dfrac{\sin 35}{\cos 35} \cdot \dfrac{1}{\sin 35} \cdot \dfrac{\cos 35}{1}$. We can cross cancel the sines. Then the cosines also cancel and we get 1 as the final answer.