

A.5 Synthetic Division

Synthetic division is alternative to long division. It is used when dividing by something in the form $x -$ or $x +$ something. In all the problems in this section there will be a one in front of the x .

EXAMPLE: Divide using synthetic division: $(x^3 - 7x^2 - 13x + 15) \div (x + 2)$

In order to do this we must first set it up. In the problem above we have $x + 2$. You want to reverse the sign of the number after the x and place it in a half box like shown below. In this case we have a negative 2 since we reversed the sign. After this number you want to write the coefficients of your equation afterwards. A coefficient is just the number that comes in front of the variable. In synthetic division there are no variables. You are only working with numbers. Under this line skip a space and then draw a line. Your set up should look like this:

$$\begin{array}{r|l} -2 & 1 \quad -7 \quad -13 \quad 15 \\ \hline & \end{array}$$

Here is how you do synthetic division. You always drop down the number right after the box like the following:

$$\begin{array}{r|l} -2 & 1 \quad -7 \quad -13 \quad 15 \\ \hline & 1 \end{array}$$

Now we will multiply the 1 by -2 and we will write this result under the -7.

$$\begin{array}{r|l} -2 & 1 \quad -7 \quad -13 \quad 15 \\ & -2 \\ \hline & 1 \end{array}$$

In synthetic division we always add. We will add -7 and -2 to get -9.

$$\begin{array}{r|l} -2 & 1 \quad -7 \quad -13 \quad 15 \\ & -2 \\ \hline & 1 \quad -9 \end{array}$$

Now multiply -9 by -2 to get 18. This number gets written under the -13.

$$\begin{array}{r|l} -2 & 1 \quad -7 \quad -13 \quad 15 \\ & -2 \quad 18 \\ \hline & 1 \quad -9 \end{array}$$

Now add -13 and 18 to get 5.

$$\begin{array}{r|l} -2 & 1 \quad -7 \quad -13 \quad 15 \\ & -2 \quad 18 \\ \hline & 1 \quad -9 \quad 5 \end{array}$$

Multiply 5 by -2 to get -10. Now write this under the 15.

$$\begin{array}{r|l} -2 & 1 \quad -7 \quad -13 \quad 15 \\ & -2 \quad 18 \quad -10 \\ \hline & 1 \quad -9 \quad 5 \end{array}$$

Now add 15 and -10 to get -5.

$$\begin{array}{r|l} -2 & 1 \quad -7 \quad -13 \quad 15 \\ & -2 \quad 18 \quad -10 \\ \hline & 1 \quad -9 \quad 5 \quad 5 \end{array}$$

Now we need to write our answer (see next page).

$$\begin{array}{r} -2 \overline{) 1 \ -7 \ -13 \ 15} \\ \underline{-2 \ 18 \ -10} \\ 1 \ -9 \ 5 \ 5 \end{array}$$

The last line above, which shows 1 -9 5 5 are the coefficients for our answer. The last 5 is our remainder, which we write over $x + 2$. We always start our answer with a power one less than what we began with. Since the original started with x^3 then our answer will start with x^2 . Write our answer as: $x^2 - 9x + 5 + \frac{5}{x+2}$.

EXAMPLE: Divide using synthetic division: $(3x^2 - 2x^3 + x^4 - 4) \div (x - 3)$

Before we set this problem up we need to make sure we have descending powers. We also need to make sure no powers are missing. If they are then we need to insert a zero as a place keeper. We will rewrite this as: $(x^4 - 2x^3 + 3x^2 + 0x - 4) \div (x - 3)$. This one the x term was missing so this is a zero place keeper.

In the problem above we have $x - 3$. You want to reverse the sign of the number after the x and place it in a half box like shown below. In this case we have a positive 3 since we reversed the sign. After this number you want to write the coefficients of your equation afterwards. Under this line skip a space and then draw a line. Notice below we still wrote the 0 to hold the place. Your set up should look like this:

$$\begin{array}{r} \underline{3} \overline{) 1 \ -2 \ 3 \ 0 \ -4} \\ \underline{ 1} \end{array} \quad \text{Start by dropping the 1 below the line we just drew.}$$

$$\begin{array}{r} \underline{3} \overline{) 1 \ -2 \ 3 \ 0 \ -4} \\ \underline{ 1} \end{array} \quad \text{Now multiply 1 by 3 and write this result under -2.}$$

$$\begin{array}{r} \underline{3} \overline{) 1 \ -2 \ 3 \ 0 \ -4} \\ \underline{ 1 } } \\ 1 \end{array} \quad \text{Now ADD the -2 and 3 to get 1. We always add with synthetic division.}$$

$$\begin{array}{r} \underline{3} \overline{) 1 \ -2 \ 3 \ 0 \ -4} \\ \underline{ 1 } } \\ 1 \end{array} \quad \text{Now multiply the 1 by 3 and write this under the 3.}$$

$$\begin{array}{r} \underline{3} \overline{) 1 \ -2 \ 3 \ 0 \ -4} \\ \underline{ 1 } } \\ 1 \end{array} \quad \text{Now multiply 6 by 3 to get 18. Write this under 0.}$$

$$\begin{array}{r} \underline{3} \overline{) 1 \ -2 \ 3 \ 0 \ -4} \\ \underline{ 1 } } \\ 1 \end{array} \quad \text{Add and repeat the process to the end. You will get:}$$

$$\begin{array}{r} \underline{3} \overline{) 1 \ -2 \ 3 \ 0 \ -4} \\ \underline{ 1 } } \\ 1 \end{array} \quad \text{The last line shows the coefficients for our answer. We always start our answer with a power one less than what we began with. Since the original started with } x^4 \text{ then our answer will start with } x^3 \text{.}$$

The 50 above is our remainder, which we write over $x - 2$. Write our answer as: $x^3 + x^2 + 6x + 18 + \frac{50}{x-3}$.

EXAMPLE: Divide using synthetic division: $(2x^5 - 6x^3 - 14x) \div (x - 2)$

We also need to make sure no powers are missing. If they are then we need to insert a zero as a place keeper. We will rewrite this as: $(2x^5 + 0x^4 - 6x^3 + 0x^2 - 14x + 0) \div (x - 2)$. We had the x^4 , x^2 and the constant term missing. You need all three of these zeros as place keepers. We are dividing by $x - 2$. You want to reverse the sign of the number after the x and place it in a half box like shown below. In this case we have a positive 2 since we reversed the sign. Your set up should look like this:

$$\begin{array}{r} \underline{2} \mid 2 \ 0 \ -6 \ 0 \ -14 \ 0 \\ \hline \end{array}$$

Start by dropping the 2 below the line we just drew.

$$\begin{array}{r} \underline{2} \mid 2 \ 0 \ -6 \ 0 \ -14 \ 0 \\ \hline 2 \end{array}$$

Multiply the 2 by 2 and write this under the 0.

$$\begin{array}{r} \underline{2} \mid 2 \ 0 \ -6 \ 0 \ -14 \ 0 \\ \hline 4 \end{array}$$

Add 0 and 4 to get 4. Now multiply the 4 by 2 to get 8. Write this under -6.

$$\begin{array}{r} \underline{2} \mid 2 \ 0 \ -6 \ 0 \ -14 \ 0 \\ \hline 4 \ 8 \end{array}$$

Add -6 and 8 to get 2. Then multiply the 2 by 2 to get 4. Write this under the 0.

$$\begin{array}{r} \underline{2} \mid 2 \ 0 \ -6 \ 0 \ -14 \ 0 \\ \hline 4 \ 8 \ 4 \end{array}$$

Add 0 and 4 to get 4. Now multiply the 4 by 2 to get 8. Write this under -14.

$$\begin{array}{r} \underline{2} \mid 2 \ 0 \ -6 \ 0 \ -14 \ 0 \\ \hline 4 \ 8 \ 4 \ 8 \end{array}$$

Add -14 and 8 to get -6. Now multiply the -6 by 2 to get -12. Write this under 0.

$$\begin{array}{r} \underline{2} \mid 2 \ 0 \ -6 \ 0 \ -14 \ 0 \\ \hline 4 \ 8 \ 4 \ 8 \ -12 \end{array}$$

Now add 0 to -12 to get -12. This is our remainder.

$$\begin{array}{r} \underline{2} \mid 2 \ 0 \ -6 \ 0 \ -14 \ 0 \\ \hline 4 \ 8 \ 4 \ 8 \ -12 \\ \hline 2 \ 4 \ 2 \ 4 \ -6 \ -12 \end{array}$$

Now we need to write our answer. The last line shows the coefficients for our answer. We always start our answer with a power one less than what we began with. Since the original started with x^5 then our answer will start with x^4 . The -12 above is our remainder, which we write over $x - 2$. Write our answer as:

$$2x^4 + 4x^3 + 2x^2 + 4x - 6 + \frac{-12}{x - 2}$$