Test C

Show all of your work. Each problem is worth 3 points. (You get 1 point for free!)

For #1.) – #24.), simplify the expressions. Write all answers using radical notation, if appropriate.
If any answer is not real, state so. Be sure to rationalize all denominators.

For #1.) – #4.), **use absolute value signs** only when necessary and appropriate.

#1.) $\sqrt[3]{\frac{27}{64y^6}}$

$= \frac{\sqrt[3]{27}}{\sqrt[3]{64y^6}} = \boxed{\frac{3}{4y^2}}$

#2.) $\sqrt{64x^6}$

$= \boxed{8|x^3|}$

#3.) $\sqrt{\frac{49}{100y^4}}$

$= \frac{\sqrt{49}}{\sqrt{100y^4}}$

$= \boxed{\frac{7}{10y^2}}$

#4.) $\sqrt[3]{-343y^{11}}$

$= \boxed{-7y^3\sqrt[3]{y^2}}$

For #5.) – #22.), assume that all variables represent positive real numbers. No absolute value is needed.

#5.) $\sqrt[3]{-40s^8t^{15}}$

$= \sqrt[3]{-8}\sqrt[3]{5s^8t^{15}}$

$= \boxed{-2s^2t^5\sqrt[3]{5s^2}}$

#6.) $3\sqrt{50} + \sqrt{8}$

$= 3\sqrt{25}\sqrt{2} + \sqrt{4}\sqrt{2}$

$= 3\cdot 5\sqrt{2} + 2\sqrt{2}$

$= 15\sqrt{2} + 2\sqrt{2}$

$= \boxed{17\sqrt{2}}$

#7.) $\sqrt[3]{-\frac{8}{x^{27}}}$

$= \frac{\sqrt[3]{-8}}{\sqrt[3]{x^{27}}}$

$= \boxed{\frac{-2}{x^9}}$

#8.) $(1+\sqrt{2})(3-\sqrt{8})$ FOIL

$3 - \sqrt{8} + 3\sqrt{2} - \sqrt{16}$

$3 - \sqrt{4}\sqrt{2} + 3\sqrt{2} - 4$

$3 - 2\sqrt{2} + 3\sqrt{2} - 4$

$\boxed{-1 + \sqrt{2}}$

#9.) $\sqrt[4]{-81}$

$\boxed{\text{not real}}$

(even index with negative radicand)

#10.) $\sqrt[4]{\frac{16}{x^{16}}}$

$= \frac{\sqrt[4]{16}}{\sqrt[4]{x^{16}}}$

$= \boxed{\frac{2}{x^4}}$

#11.) $\frac{6}{(3-\sqrt{7})}\cdot\frac{(3+\sqrt{7})}{(3+\sqrt{7})}$

$= \frac{6(3+\sqrt{7})}{9-7} = \frac{3\cancel{6}(3+\sqrt{7})}{\cancel{12}}$

$= \boxed{3(3+\sqrt{7})}$

#12.) $\sqrt{72a^7b^9}$

$= \sqrt{36}\sqrt{2a^7b^9}$

$= \boxed{6a^3b^4\sqrt{2ab}}$

#13.) $-25^{-\frac{3}{2}}$

$= -\frac{1}{25^{3/2}} = -\frac{1}{(\sqrt{25})^3}$

$= -\frac{1}{(5)^3} = \boxed{-\frac{1}{125}}$

#14.) $(\sqrt{2} - \sqrt{7})^2$

$= (\sqrt{2} - \sqrt{7})(\sqrt{2} - \sqrt{7})$

$= \sqrt{4} - \sqrt{14} - \sqrt{14} + \sqrt{49}$

$= 2 - 2\sqrt{14} + 7$

$= \boxed{9 - 2\sqrt{14}}$

#15.) $(-49)^{\frac{3}{2}}$

$= \left(\sqrt{-49}\right)^3$

$= \boxed{\text{not real}}$

#16.) $\dfrac{\sqrt[2]{y}}{\sqrt[3]{y^2}} \cdot \dfrac{\sqrt[3]{y}}{\sqrt[3]{y}}$

$= \dfrac{\sqrt[3]{y^2}}{\sqrt[3]{y^3}} = \boxed{\dfrac{\sqrt[3]{y^2}}{y}}$

#17.) $\sqrt{12x} + \sqrt{27x^3}$

$= \sqrt{4}\sqrt{3x} + \sqrt{9}\sqrt{3x^3}$

$= 2\sqrt{3x} + 3x\sqrt{3x}$

$= \boxed{(2 + 3x)\sqrt{3x}}$

#18.) $3\sqrt{48} + 5\sqrt{27}$

$= 3\sqrt{16}\sqrt{3} + 5\sqrt{9}\sqrt{3}$

$= 3 \cdot 4\sqrt{3} + 5 \cdot 3\sqrt{3}$

$= 12\sqrt{3} + 15\sqrt{3}$

$= \boxed{27\sqrt{3}}$

#19.) $\sqrt[4]{48x^3y^{12}}$

$= \sqrt[4]{16}\sqrt[4]{3x^3y^{12}}$

$= \boxed{2y^3\sqrt[4]{3x^3}}$

#20.) $\dfrac{4}{(\sqrt{5} + \sqrt{3})} \cdot \dfrac{(\sqrt{5} - \sqrt{3})}{(\sqrt{5} - \sqrt{3})}$

$= \dfrac{4(\sqrt{5} - \sqrt{3})}{5 - 3}$

$= \dfrac{\cancel{4}(\sqrt{5} - \sqrt{3})}{\cancel{2}}$

$= \boxed{2(\sqrt{5} - \sqrt{3})}$

#21.) $\sqrt[3]{125x^4} - x\sqrt[3]{343x}$

$5x\sqrt[3]{x} - 7x\sqrt[3]{x}$

$\boxed{-2x\sqrt[3]{x}}$

#22.) $\dfrac{5}{\sqrt{50}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{5\sqrt{2}}{\sqrt{100}} = \dfrac{15\sqrt{2}}{\cancel{20}}$

$= \boxed{\dfrac{\sqrt{2}}{2}}$

#23.) Find the distance between the points $(-5, 3)$ and $(-1, 7)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{((-5) - (-1))^2 + ((3) - (7))^2}$

$d = \sqrt{(-4)^2 + (-4)^2}$

$d = \sqrt{16 + 16}$

$d = \sqrt{32} = \sqrt{16}\sqrt{2}$

$d = \boxed{4\sqrt{2}}$

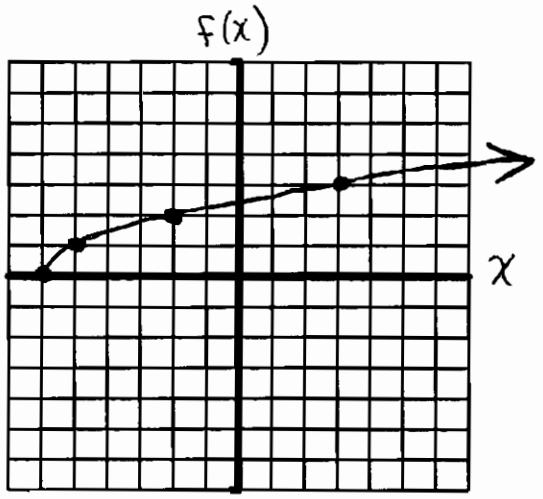
#24.) Graph the function and give the domain and range:

$f(x) = \sqrt{x + 6}$ (Hint: Make a table of values)

x	$f(x)$
-6	0
-5	1
-2	2
3	3

$D: [-6, \infty)$

$R: [0, \infty)$



For #25.) – #28.), solve the equation. Be sure to check your answers.

#25.) $5 = \sqrt{5r - 4} - 4$

$(9)^2 = \left(\sqrt{5r-4}\right)^2$

$81 = 5r - 4$

$85 = 5r$

$17 = r$

$\{17\}$

#26.) $\sqrt[4]{3k + 2} + 2 = 0$

$\left(\sqrt[4]{3k+2}\right)^4 = (-2)^4$

$3k + 2 = 16$

$3k = 14$

$k = \frac{14}{3}$ ← doesn't check!

$\emptyset$

#27.) $\sqrt{2x + 1} + 1 = x$

$\left(\sqrt{2x+1}\right)^2 = (x-1)^2$

$2x + 1 = x^2 - 2x + 1$

$0 = x^2 - 4x$

$0 = x(x - 4)$

$\{4\}$

$x = 0$ doesn't check!

$x - 4 = 0$

$x = 4$

#28.) $\left(\sqrt{x - 5}\right)^2 = \left(\sqrt{x} - 1\right)^2$ FOIL

$x - 5 = x - 2\sqrt{x} + 1$

$\frac{-6}{-2} = \frac{-2\sqrt{x}}{-2}$

$(3)^2 = \left(\sqrt{x}\right)^2$

$9 = x$

$\{9\}$

For #29.) – #32.), perform the indicated operation. Use the number "i" when appropriate

#29.) $(5 + \sqrt{-16}) - (2 - \sqrt{-36})$

$(5 + 4i) - (2 - 6i)$

$5 + 4i - 2 + 6i$

$3 + 10i$

#30.) Simplify: i^{27} $27 \div 4 = 6$ R 3

$i^{27} = i^3 = -i$

#31.) $(2 - i)(3 + 4i)$

$6 + 8i - 3i - 4i^2$

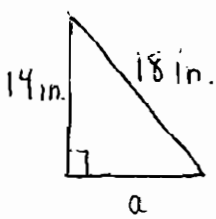
$6 + 5i + 4(+1)$

$10 + 5i$

#32.) $\frac{6i}{(1+i)} \cdot \frac{(1-i)}{(1-i)} = \frac{6i - 6i^2}{1 - i^2} = \frac{6i - 6(-1)}{1 - (-1)}$

$= \frac{6 + 6i}{2} = \frac{2(3+3i)}{2} = 3 + 3i$

#33.) The hypotenuse of a right triangle measures 18 in and one leg measures 14 in. Find the exact (not a decimal) length of the other leg of the triangle.



$a^2 + b^2 = c^2$

$a^2 + 14^2 = 18^2$

$a^2 + 196 = 324$

$a^2 = 128$

$a = \sqrt{128}$

$a = \sqrt{64}\sqrt{2}$

$a = 8\sqrt{2}$ in.