Handout # 5.1 Trigonometric identities:

$$\sin\alpha\ \cos\beta = \frac{1}{2}\sin(\alpha-\beta)+\frac{1}{2}\sin(\alpha+\beta)$$

$$\cos\alpha\ \cos\beta = \frac{1}{2}\cos(\alpha-\beta)+\frac{1}{2}\cos(\alpha+\beta)$$

$$\sin\alpha\ \sin\beta = \frac{1}{2}\cos(\alpha-\beta)-\frac{1}{2}\cos(\alpha+\beta)$$

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2} \qquad \sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$\sin(\alpha \pm \beta) = \sin\alpha\ \cos\beta \pm \cos\alpha\ \sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\ \cos\beta \mp \sin\alpha\ \sin\beta$$

$$\sin 2\alpha = 2\sin\alpha\ \cos\alpha \qquad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\tan(\alpha\pm\beta) = \frac{\tan\alpha \pm \tan\beta}{1\mp\tan\alpha\tan\beta} \qquad \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\sin\alpha \pm \sin\beta = 2\sin\left(\frac{\alpha\pm\beta}{2}\right)\cos\left(\frac{\alpha\mp\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)$$

How to exit Trigonometry:

$$\tan\frac{x}{2} = y \Rightarrow \sin x = \frac{2y}{1+y^2}\ ,\ \cos x = \frac{1-y^2}{1+y^2}\ ,\ dx = \frac{2dy}{1+y^2}$$

$$\tan x = y \Rightarrow \cos^2 x = \frac{1}{1+y^2}\ ,\ \sin^2 x = \frac{y^2}{1+y^2}\ ,\ \sin x\cos x = \frac{y}{1+y^2}\ ,\ dx = \frac{dy}{1+y^2}$$