

Handout # 6**Power Series for Elementary Functions**

$$1) \quad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad -1 < x < 1$$

$$2) \quad e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad x \in (-\infty, +\infty)$$

$$3) \quad \sin x = \sum_{n=0}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!} \quad x \in (-\infty, +\infty)$$

$$4) \quad \cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \quad x \in (-\infty, +\infty)$$

$$5) \quad \ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \quad -1 < x \leq 1$$

$$6) \quad \arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \quad -1 \leq x \leq 1$$

$$7) \quad (1+x)^{\alpha} = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n \quad \binom{\alpha}{0} = 1, \quad \binom{\alpha}{1} = \alpha, \dots,$$

$$\binom{\alpha}{n} = \frac{\alpha(\alpha-1)(\alpha-2)\dots(\alpha-(n-1))}{n!}, \quad -1 < x < 1 \quad (\alpha \neq \text{integer})$$

$$8) \quad \arcsin x = \sum_{n=0}^{\infty} \frac{(2n-1)!!}{(2n)!!} \cdot \frac{x^{2n+1}}{2n+1} \quad -1 \leq x \leq 1$$

$(2n-1)!! = (1)(3)(5)\dots(2n-1)$, $(2n)!! = (2)(4)(6)\dots(2n)$, $1!! = 1$, $2!! = 2$, $0!! = 1$, $(-1)!! = 1$