# Math 96--Radicals #1--
# Simplify; Combine--page 1

**Part A–Number Systems**

a. Whole Numbers = {0, 1, 2, 3, ...}

b. Integers = whole numbers and their opposites = {..., −3, −2, −1, 0, 1, 2, 3, ...}

c. Rational Numbers = quotient of integers (the decimal equivalents of rational numbers terminate or repeat)

examples: $\frac{3}{5}$ (quotient of integers, which equals 0.6, which is a terminating decimal)

$\frac{3}{11}$ (quotient of integers, which equals 0.272727..., which is a repeating decimal)

9 (which equals $\frac{9}{1}$, quotient of integers)

−6 (which equals $\frac{-6}{1}$, quotient of integers)

$\sqrt{64}$ = 8 (8 = $\frac{8}{1}$, quotient of integers)

d. Irrational Numbers = numbers which cannot be written as the quotient of integers (the decimal equivalents of irrational numbers never end and never repeat)

examples: $\pi$ $\sqrt{2}$ $\sqrt{10}$ $\sqrt{12} = 2\sqrt{3}$

The decimal equivalents of these numbers never end and never repeat.

e. Real Numbers = rational numbers and irrational numbers

State the type of number:

f. 5 is whole, integer, rational $\left(\frac{5}{1}\right)$, real

g. −8 is integer, rational, real

h. $\frac{3}{8}$ is rational, real

i. $\sqrt{15}$ is irrational, real

**Homework.** State the type of number.

1. $\frac{7}{20}$

2. 19

3. a. $\sqrt{20}$
   b. $\sqrt{49}$

4. −11

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**Radical Expression–**You will be working with radicals expressions: $\sqrt[n]{a}$ read "the nth root of a" where n is the index and a is the radicand. For example, $\sqrt[5]{2}$ is read "the 5th root of 2" where 5 is the index and 2 is the radicand.

**Part B–Memorizing Common Square Roots–**There are certain "perfect" square roots that you should know by memory. If you don't know these by memory, then you'll have to use the technique described under the next section. Here are the common "perfect" square roots you should know.

| | | | | |
|---|---|---|---|---|
| $\sqrt{0}=0$ | $\sqrt{1}=1$ | $\sqrt{4}=2$ | $\sqrt{9}=3$ | $\sqrt{16}=4$ |
| $\sqrt{25}=5$ | $\sqrt{36}=6$ | $\sqrt{49}=7$ | $\sqrt{64}=8$ | $\sqrt{81}=9$ |
| $\sqrt{100}=10$ | $\sqrt{121}=11$ | $\sqrt{144}=12$ | | |

**Part C–Simplifying Radicals**–Here's the first rule of radicals we'll be using: You cannot leave a "perfect" or a "perfect" factor underneath the radical symbol. The index tells you how many identical factors you need to make a "perfect" to take to the outside of the radical symbol. The non-perfect factors are left underneath the radical symbol. See the examples.

j. $\sqrt{\Delta^6} = \sqrt{\Delta\cdot\Delta\cdot\Delta\cdot\Delta\cdot\Delta\cdot\Delta} = \Delta\cdot\Delta\cdot\Delta = \Delta^3$

k. $\sqrt[3]{\Delta^6} = \sqrt[3]{\Delta\cdot\Delta\cdot\Delta\cdot\Delta\cdot\cdot\Delta\cdot\Delta} = \Delta\cdot\Delta = \Delta^2$

l. $\sqrt{x^2} = \sqrt{xx} = x$
$\sqrt{x^4} = \sqrt{xx\ xx} = x\,x = x^2$
$\sqrt{x^6} = \sqrt{xx\ xx\ xx} = x\,x\,x = x^3$

m. $\sqrt[3]{x^3} = \sqrt[3]{xxx} = x$
$\sqrt[3]{x^6} = \sqrt[3]{xxx\ xxx} = x\,x = x^2$
$\sqrt[3]{x^9} = \sqrt[3]{xxx\ xxx\ xxx} = x\,x\,x = x^3$

n. $\sqrt{x^4\ y^6} = \sqrt{xx\,xx\,yy\,yy\,yy} = x\,x\,y\,y\,y = x^2y^3$

o. $\sqrt[3]{a^3b^6} = \sqrt[3]{aaa\,bbb\,bbb} = a\,b\,b = ab^2$

p. $\sqrt[4]{x^4} = \sqrt[4]{xxxx} = x$
$\sqrt[4]{x^8} = \sqrt[4]{xxxx\ xxxx} = x^2$
$\sqrt[4]{x^{12}} = \sqrt[4]{xxxx\ xxxx\ xxxx} = x^3$

q. $\sqrt{x^5} = \sqrt{xx\ xx\ x} = x\,x\sqrt{x} = x^2\sqrt{x}$
$\sqrt[3]{x^4} = \sqrt[3]{xxx\ x} = x\sqrt[3]{x}$
$\sqrt[3]{x^5} = \sqrt[3]{xxx\ xx} = x\sqrt[3]{xx} = x\sqrt[3]{x^2}$

r. $\sqrt{25} = \sqrt{5\cdot 5} = 5$

s. $\sqrt[3]{8} = \sqrt[3]{2\cdot 2\cdot 2} = 2$

t. $\sqrt{12} = \sqrt{2\cdot 2\cdot 3} = 2\sqrt{3}$

u. $\sqrt[3]{16} = \sqrt[3]{2\cdot 2\cdot 2\cdot 2} = 2\sqrt[3]{2}$

v. $\sqrt[4]{405} = \sqrt[4]{3\cdot 3\cdot 3\cdot 3\cdot 5} = 3\sqrt[4]{5}$

w. $\sqrt[3]{48} = \sqrt[3]{2\cdot 2\cdot 2\cdot 2\cdot 3} = 2\sqrt[3]{2\cdot 3} = 2\sqrt[3]{6}$

**HOMEWORK.** Simplify the following.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 5. | $\sqrt{m^8}$ | 6. | $\sqrt{z^{10}}$ | 7. | $\sqrt[3]{a^{12}}$ | 8. | $\sqrt[3]{j^9k^{15}}$ |
| 9. | $\sqrt{x^7}$ | 10. | $\sqrt{a^5b^9}$ | 11. | $\sqrt[3]{k^7}$ | 12. | $\sqrt[3]{y^{10}z^5}$ |
| 13. | $\sqrt{100}$ | 14. | $\sqrt{81}$ | 15. | $\sqrt[3]{64}$ | 16. | $\sqrt[3]{125}$ |
| 17. | $\sqrt[3]{216}$ | 18. | $\sqrt[3]{343}$ | 19. | $\sqrt[4]{81}$ | 20. | $\sqrt[4]{625}$ |

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| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 21. | $\sqrt[5]{32}$ | 22. | $\sqrt{12}$ | 23. | $\sqrt{20}$ | 24. | $\sqrt{24}$ |
| 25. | $\sqrt{28}$ | 26. | $\sqrt{18}$ | 27. | $\sqrt{27}$ | 28. | $\sqrt{45}$ |
| 29. | $\sqrt{32}$ | 30. | $\sqrt{48}$ | 31. | $\sqrt{50}$ | 32. | $\sqrt{75}$ |
| 33. | $\sqrt{125}$ | 34. | $\sqrt[3]{16}$ | 35. | $\sqrt[3]{54}$ | 36. | $\sqrt[4]{162}$ |
| 37. | $\sqrt[5]{96}$ | 38. | $\sqrt{108}$ | 39. | $\sqrt[3]{432}$ | 40. | $\sqrt[3]{40}$ |

By the way, there are alternatives to factoring completely using primes. If you recognize a "perfect" square root, you can use that instead. For example, on $\sqrt{72}$ , you could factor this way: $\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$ . This would require you to recognize the perfect square root 36. I usually use prime factors unless I quickly recognize a "perfect". As another alternative, you could factor $\sqrt{72}$ in this way: $\sqrt{72} = \sqrt{9 \cdot 8} = 3\sqrt{8} = 3\sqrt{4 \cdot 2} = 3 \cdot 2\sqrt{2} = 6\sqrt{2}$ . Compare both the above to the prime factoring: $\sqrt{72} = \sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3} = 2 \cdot 3\sqrt{2} = 6\sqrt{2}$

Notice that all three ways give you the same result. You just have to practice enough to be comfortable with the concept. You also have to completely simplify so you may have to continue factoring if you use an alternative method to prime factors.

**Part D–More Simplifying**–Sometimes you will have simplifying radicals that combines ideas. See the example.

x. $\sqrt{12x^5} = \sqrt{2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x} = 2 \cdot x \cdot x \cdot \sqrt{3 \cdot x} = 2x^2\sqrt{3x}$

**Homework.** Simplify the following.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 41. | $\sqrt{36m^2}$ | 42. | $\sqrt{49y^3}$ | 43. | $\sqrt{45k^6}$ | 44. | $\sqrt{18a^7}$ |

**Part E–Negative Radicands**–When you have a negative radicand with an even index, you have no result. When you have a negative radicand with an odd index, the result is negative. Generally, on the odd index problems, I immediately take the negative to the "outside" (just to save time because a negative to an odd power remains a negative), factor the radicand, and continue the process as above. See the examples.

y. $\sqrt{-4}$ = no result

z. $\sqrt[3]{-8} = -\sqrt[3]{8} = -\sqrt[3]{2 \cdot 2 \cdot 2} = -2$ or
$\sqrt[3]{-8} = \sqrt[3]{(-2)(-2)(-2)} = -2$

aa. $\sqrt[4]{-16}$ = no result

bb. $\sqrt[5]{-243} = -\sqrt[5]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = -3$ or
$\sqrt[5]{-243} = \sqrt[5]{(-3)(-3)(-3)(-3)(-3)} = -3$

**HOMEWORK.** Simplify the following.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 45. | $\sqrt{-25}$ | 46. | $\sqrt[4]{-81}$ | 47. | $\sqrt[3]{-27}$ | 48. | $\sqrt[5]{-32}$ |

**Part F–Factoring Trinomials**–There are times when the radicand contains phrases. Observe the following.

cc. $\sqrt{(x+2)(x+2)} = x + 2$

dd. $\sqrt{(m-4)(m-4)} = m - 4$

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Sometimes the radicand is a trinomial, which must first be factored. Observe.

ee. $\sqrt{y^2+6y+9} = \sqrt{(y+3)(y+3)} = y+3$

ff. $\sqrt{k^2-2k+1} = \sqrt{(k-1)(k-1)} = k-1$

**HOMEWORK.** Simplify.

49. $\sqrt{z^2+10z+25}$

50. $\sqrt{a^2-16a+64}$

**Part G–Fractions and Radicals–**When you have a fraction under a radical, you can re-write the fraction with the radical on the numerator and the radical on the denominator. Then use the rules from above to simplify. Observe.

gg. $\sqrt{\frac{16}{49}} = \frac{\sqrt{16}}{\sqrt{49}} = \frac{4}{7}$

hh. $\sqrt{\frac{18}{25}} = \frac{\sqrt{18}}{\sqrt{25}} = \frac{\sqrt{2\cdot 3\cdot 3}}{5} = \frac{3\sqrt{2}}{5}$

**HOMEWORK.** Simplify.

51. $\sqrt{\frac{9}{100}}$

52. $\sqrt{\frac{50}{36}}$

**Part H–Combining Like Radicals**–Now we're ready for combining like radicals. Add/subtract like terms where necessary. When the index and the radicand are both identical, you can combine the coefficients of the like terms. Sometimes, the radicands do not look the same to begin with, but after factoring, the radicands may then be the same.

ii. $5\sqrt{7}+8\sqrt{7}-2\sqrt{7}$ — All square roots; all radicands are 7; so combine coefficients

$11\sqrt{7}$

jj. $6\sqrt[3]{2}-17\sqrt[3]{2}+7\sqrt[3]{2}$ — All cube roots; all radicands are 2; so combine coefficients

$-4\sqrt[3]{2}$

kk. $7\sqrt{12}+6\sqrt{27}$ — Begin by factoring

$7\sqrt{2\cdot 2\cdot 3}+6\sqrt{3\cdot 3\cdot 3}$

$7\cdot 2\sqrt{3}+6\cdot 3\sqrt{3}$

$14\sqrt{3}+18\sqrt{3}$

$32\sqrt{3}$

ll. $7\sqrt[3]{54}+9\sqrt[3]{2}-5\sqrt[3]{250}$

$7\sqrt[3]{2\cdot 3\cdot 3\cdot 3}+9\sqrt[3]{2}-5\sqrt[3]{2\cdot 5\cdot 5\cdot 5}$

$21\sqrt[3]{2}+9\sqrt[3]{2}-25\sqrt[3]{2}$

$5\sqrt[3]{2}$

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**HOMEWORK.** Simplify the following.

| | | | |
|---|---|---|---|
| 53. | $5\sqrt{2} + 8\sqrt{2} - 11\sqrt{2}$ | 54. | $4\sqrt{18} + 8\sqrt{50}$ |
| 55. | $9\sqrt{5} - 2\sqrt{45} + 6\sqrt{20}$ | 56. | $\sqrt{28} + 3\sqrt{63}$ |
| 57. | $9\sqrt{12} + 7\sqrt{27} - 2\sqrt{48} - 3\sqrt{75}$ | 58. | $2\sqrt[3]{56} - 11\sqrt[3]{7} - 4\sqrt[3]{189}$ |
| 59. | $6\sqrt[3]{24} - 10\sqrt[3]{81} + 5\sqrt[3]{3}$ | 60. | $14\sqrt[4]{5} + 6\sqrt[4]{80} + 7\sqrt[4]{405}$ |

**Answer Key.**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | rational<br>real | 2. | whole<br>integer<br>rational<br>real | 3a.<br><br>3b. | irrational<br>real<br>whole, integer<br>rational, real | 4. | integer<br>rational<br>real |
| 5. | $m^4$ | 6. | $z^5$ | 7. | $a^4$ | 8. | $j^3$ k5 |
| 9. | $x^3\sqrt{x}$ | 10. | $a^2b^4\sqrt{ab}$ | 11. | $k^2\sqrt[3]{k}$ | 12. | $y^3z\sqrt[3]{yz^2}$ |
| 13. | 10 | 14. | 9 | 15. | 4 | 16. | 5 |
| 17. | 6 | 18. | 7 | 19. | 3 | 20. | 5 |
| 21. | 2 | 22. | $2\sqrt{3}$ | 23. | $2\sqrt{5}$ | 24. | $2\sqrt{6}$ |
| 25. | $2\sqrt{7}$ | 26. | $3\sqrt{2}$ | 27. | $3\sqrt{3}$ | 28. | $3\sqrt{5}$ |
| 29. | $4\sqrt{2}$ | 30. | $4\sqrt{3}$ | 31. | $5\sqrt{2}$ | 32. | $5\sqrt{3}$ |
| 33. | $5\sqrt{5}$ | 34. | $2\sqrt[3]{2}$ | 35. | $3\sqrt[3]{2}$ | 36. | $3\sqrt[4]{2}$ |
| 37. | $2\sqrt[5]{3}$ | 38. | $6\sqrt{3}$ | 39. | $6\sqrt[3]{2}$ | 40. | $2\sqrt[3]{5}$ |
| 41. | 6m | 42. | $7y\sqrt{y}$ | 43. | $3k^3\sqrt{5}$ | 44. | $3a^3\sqrt{2a}$ |
| 45. | no result | 46. | no result | 47. | –3 | 48. | –2 |
| 49. | z + 5 | 50. | a – 8 | 51. | $\frac{3}{10}$ | 52. | $\frac{5\sqrt{2}}{6}$ |
| 53. | $2\sqrt{2}$ | 54. | $52\sqrt{2}$ | 55. | $15\sqrt{5}$ | 56. | $11\sqrt{7}$ |
| 57. | $16\sqrt{3}$ | 58. | $-19\sqrt[3]{7}$ | 59. | $-13\sqrt[3]{3}$ | 60. | $47\sqrt[4]{5}$ |