

## Math 95--Equations with Fractions--page 1

How do you solve equations when the equation itself contains fractions? One approach to solving equations involving fractions is to **clear the equation of fractions**. This means you multiply ALL terms by the same LCD, eliminating all denominators. The numerators will change accordingly. The new equation will look different but will have the same numerical value. Then solve, using all the methods you know. Observe.

a.  $\frac{3}{8}x + \frac{5}{6} = \frac{2}{3}$  multiply all terms by LCD 24

$$24\left(\frac{3}{8}x\right) + 24\left(\frac{5}{6}\right) = 24\left(\frac{2}{3}\right)$$

reduce 24 with each denominator

$$3(3x) + 4(5) = 8(2)$$

now multiply to remove parentheses

$$9x + 20 = 16$$
$$9x + 20 - \mathbf{20} = 16 - \mathbf{20}$$
$$9x = -4$$
$$9x \div \mathbf{9} = -4 \div \mathbf{9}$$
$$x = -\frac{4}{9}$$

Suppose you want to check a result. Observe the check for example a.

$$\frac{3}{8}x + \frac{5}{6} = \frac{2}{3}$$

check  $x = -\frac{4}{9}$

$$\frac{3}{8}\left(-\frac{4}{9}\right) + \frac{5}{6} = \frac{2}{3}$$
$$-\frac{12}{72} + \frac{5}{6} = \frac{2}{3}$$
$$-\frac{1}{6} + \frac{5}{6} = \frac{2}{3}$$
$$\frac{4}{6} = \frac{2}{3}$$
$$\frac{2}{3} = \frac{2}{3}$$

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b.  $\frac{5}{6}x + \frac{7}{12} = \frac{3}{8}x + \frac{4}{3}$  multiply all terms by LCD 24

$$24\left(\frac{5}{6}x\right) + 24\left(\frac{7}{12}\right) = 24\left(\frac{3}{8}x\right) + 24\left(\frac{4}{3}\right)$$

reduce 24 with each denominator

$$4(5x) + 2(7) = 3(3x) + 8(4)$$

now multiply to remove parentheses

$$20x + 14 = 9x + 32$$

$$\begin{array}{r} -9x \qquad -9x \\ \hline 11x + 14 = \qquad 32 \\ -14 \qquad -14 \\ \hline 11x \qquad = \qquad 18 \end{array}$$

$$\frac{11x}{11} = \frac{18}{11}$$

$$x = \frac{18}{11}$$

Suppose you multiply the equation in example b by the common denominator 48 (48 is a common denominator, but 24 is the lowest common denominator). You should still eventually get the same result. Observe.

$$\frac{5}{6}x + \frac{7}{12} = \frac{3}{8}x + \frac{4}{3}$$

$$48\left(\frac{5}{6}x\right) + 48\left(\frac{7}{12}\right) = 48\left(\frac{3}{8}x\right) + 48\left(\frac{4}{3}\right)$$

$$8(5x) + 4(7) = 6(3x) + 16(4)$$

$$40x + 28 = 18x + 64$$

$$\begin{array}{r} -18x \qquad -18x \\ \hline 22x + 28 = \qquad 64 \\ -28 \qquad -28 \\ \hline 22x \qquad = \qquad 36 \end{array}$$

$$\frac{22x}{22} = \frac{36}{22}$$

$$x = \frac{36}{22} = \frac{18}{11}$$

Notice the result is the same!

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c.  $3x - \frac{1}{4} = -\frac{7}{10}$

multiply all terms by LCD 20

$$20(3x) - 20\left(\frac{1}{4}\right) = 20\left(-\frac{7}{10}\right)$$

reduce 20 with each denominator

$$20(3x) - 5(1) = 2(-7)$$

now multiply to remove parentheses

$$60x - 5 = -14$$

$$\begin{array}{r} + 5 \quad + 5 \\ \hline 60x \quad = \quad -9 \end{array}$$

$$\frac{60x}{60} = \frac{-9}{60}$$

$$x = -\frac{9}{60} = -\frac{3}{20}$$

d.  $\frac{3}{8}x = 11$

multiply all terms by LCD 8

$$8\left(\frac{3}{8}x\right) = 8(11)$$

$$1(3x) = 8(11)$$

$$3x = 88$$

divide both sides by 3; you will get a fraction result

$$x = \frac{88}{3}$$

e.  $\frac{2}{3}x + 5 = 8$

multiply all terms by LCD 3

$$3\left(\frac{2}{3}x\right) + 3(5) = 3(8)$$

$$1(2x) + 3(5) = 3(8)$$

$$2x + 15 = 24$$

$$\begin{array}{r} - 15 \quad - 15 \\ \hline 2x \quad = \quad 9 \end{array}$$

$$\frac{2x}{2} = \frac{9}{2}$$

$$x = \frac{9}{2}$$

Suppose on example e, you saw the following:

$$\left(\frac{2}{3}\right)x + 5 = 8$$

remove parentheses

$$\frac{2}{3}x + 5 = 8$$

same equation as above; solve by clearing the fraction

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f.  $k + \frac{2}{3} = \frac{5}{8}$

multiply all terms by LCD 24

$$24(k) + 24\left(\frac{2}{3}\right) = 24\left(\frac{5}{8}\right)$$

$$24(k) + 8(2) = 3(5)$$

$$24k + 16 = 15$$

$$\begin{array}{r} -16 \quad -16 \\ \hline 24k \quad = -1 \end{array}$$

$$\frac{24k}{24} = \frac{-1}{24}$$

$$k = -\frac{1}{24}$$

**The only way to get really good with these problems is to practice. So you try these!** The more you do, the easier these get.

1.  $m + \frac{3}{8} = \frac{7}{12}$

2.  $y + \frac{1}{5} = -\frac{8}{15}$

3.  $k - \frac{5}{12} = -\frac{3}{10}$

4.  $\frac{3}{5}z = 7$

5.  $-\frac{2}{3}a = \frac{8}{15}$

6.  $\frac{5}{12}g + \frac{2}{3} = \frac{1}{8}$

7.  $\frac{2}{9}w - \frac{1}{4} = \frac{5}{6}$

8.  $\frac{7}{15}x - 4 = 2$

9.  $\left(\frac{3}{7}\right)x + 2 = 4$

10.  $\left(\frac{7}{9}\right)x - \frac{1}{2} = 3$

11.  $\frac{2}{3}x + \frac{5}{6} = \frac{1}{2}x - \frac{7}{9}$

12.  $\frac{3}{4}x + \frac{7}{12} = \frac{1}{9}x + \frac{5}{6}$

13.  $\frac{1}{8}x - \frac{3}{4} = \frac{7}{16}x + \frac{1}{2}$

14.  $\frac{3}{5}x - \frac{7}{10} = \frac{1}{2}x - \frac{5}{4}$

15.  $\frac{5}{8}x - \frac{1}{3} = 2x - \frac{5}{6}$

16.  $\left(\frac{2}{3}\right)x + 1 = \frac{5}{6}x + \frac{3}{4}$

17.  $2x - \frac{4}{7} = \left(\frac{1}{2}\right)x + \frac{9}{14}$

18.  $\frac{3}{11}x - 2 = 5x - \frac{1}{2}$

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19.  $-\frac{7}{8}x - \frac{5}{6} = \frac{11}{24}x - \frac{5}{16}$

20.  $\frac{5}{7}x - \frac{3}{14} = -\frac{3}{8}x + \frac{3}{4}$

21.  $\frac{1}{3}x + \frac{2}{9} = \frac{5}{6}x + \frac{1}{2}$

22.  $\frac{3}{4}x - \frac{4}{9} = \frac{7}{18}x - \frac{2}{3}$

23.  $\frac{1}{5}x + 2 = \frac{3}{8}$

24.  $\frac{3}{10}x - \frac{1}{4} = 3$

25.  $x + \frac{7}{12} = 2$

26.  $x - \frac{2}{3} = \frac{7}{6}$

27.  $\left(\frac{2}{7}\right)x - 3 = -8$

28.  $\frac{5}{12}x + \frac{1}{6} = -\frac{4}{3}$

29.  $\frac{3}{5}x = \frac{7}{10}$

30.  $\frac{3}{8}x = 6$

**Answer Key.** On the first part of the answer key, I will tell the LCD. I will show the step after reducing and the step after simplifying. All solving is left to you; the answer will then be shown. On the rest of the answer key, I will show the answers (and I might show the equation after simplifying).

1. LCD 24  
 $24(m) + 3(3) = 2(7)$   
 $24m + 9 = 14$   
 $m = \frac{5}{24}$

2. LCD 15  
 $15(y) + 3(1) = 1(-8)$   
 $15y + 3 = -8$   
 $y = -\frac{11}{15}$

3. LCD 60  
 $60(k) - 5(5) = 6(-3)$   
 $60k - 25 = -18$   
 $k = \frac{7}{60}$

4. LCD 5  
 $1(3z) = 5(7)$   
 $3z = 35$   
 $z = \frac{35}{3}$

5. LCD 15  
 $5(-2a) = 1(8)$   
 $-10a = 8$   
 $a = \frac{8}{-10} = -\frac{4}{5}$

6. LCD 24  
 $2(5g) + 8(2) = 3(1)$   
 $10g + 16 = 3$   
 $g = -\frac{13}{10}$

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7. LCD 36  
 $4(2w) - 9(1) = 6(5)$   
 $8w - 9 = 30$   
 $w = \frac{39}{8}$

8. LCD 15  
 $1(7x) - 15(4) = 15(2)$   
 $7x - 60 = 30$   
 $x = \frac{90}{7}$

9. LCD 7  
 $1(3x) + 7(2) = 7(4)$   
 $3x + 14 = 28$   
 $x = \frac{14}{3}$

10. LCD 18  
 $2(7x) - 9(1) = 18(3)$   
 $14x - 9 = 54$   
 $x = \frac{63}{14} = \frac{9}{2}$

11. LCD 18  
 $6(2x) + 3(5) = 9(1x) - 2(7)$   
 $12x + 15 = 9x - 14$   
 $x = -\frac{29}{3}$

12. LCD 36  
 $9(3x) + 3(7) = 4(1x) + 6(5)$   
 $27x + 21 = 4x + 30$   
 $x = \frac{9}{23}$

13. LCD 16  
 $2(1x) - 4(3) = 1(7x) + 8(1)$   
 $2x - 12 = 7x + 8$   
 $x = -4$

14. LCD 20  
 $4(3x) - 2(7) = 10(1x) - 5(5)$   
 $12x - 14 = 10x - 25$   
 $x = -\frac{11}{2}$

15. LCD 24  
 $3(5x) - 8(1) = 24(2x) - 4(5)$   
 $15x - 8 = 48x - 20$   
 $x = \frac{-12}{-33} = \frac{4}{11}$

16. LCD 12  
 $4(2x) + 12(1) = 2(5x) + 3(3)$   
 $8x + 12 = 10x + 9$   
 $x = \frac{3}{2}$

17. LCD 14  
 $14(2x) - 2(4) = 7(1x) + 1(9)$   
 $28x - 8 = 7x + 9$   
 $x = \frac{17}{21}$

18. LCD 22  
 $2(3x) - 22(2) = 22(5x) - 11(1)$   
 $6x - 44 = 110x - 11$   
 $x = -\frac{33}{104}$

19. LCD 48  
 $6(-7x) - 8(5) = 2(11x) - 3(5)$   
 $-42x - 40 = 22x - 15$   
 $x = -\frac{25}{64}$

20. LCD 56  
 $8(5x) - 4(3) = 7(-3x) + 14(3)$   
 $40x - 12 = -21x + 42$   
 $x = \frac{54}{61}$

21.  $6x + 4 = 15x + 9$   
 $x = \frac{5}{-9} = -\frac{5}{9}$

22.  $27x - 16 = 14x - 24$   
 $x = \frac{-8}{13} = -\frac{8}{13}$

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23.  $x = \frac{-65}{8} = -\frac{65}{8}$

24.  $x = \frac{65}{6}$

25.  $12x + 7 = 24$   
 $x = \frac{17}{12}$

26.  $6x - 4 = 7$   
 $x = \frac{11}{6}$

27.  $x = \frac{-35}{2} = -\frac{35}{2}$

28.  $x = \frac{-18}{5} = -\frac{18}{5}$

29.  $6x = 7$   
 $x = \frac{7}{6}$

30.  $3x = 48$   
 $x = 16$

**Additional Practice, if you feel you need more! The more you practice, the better you get :) )**

31.  $\frac{3}{8}x + \frac{5}{6} = \frac{5}{4}x + \frac{2}{3}$

32.  $\frac{2}{9}x - \frac{1}{3} = \frac{5}{6}x - \frac{1}{2}$

33.  $\frac{2}{5}x + \frac{1}{4} = -\frac{7}{10}$

34.  $\frac{4}{15}x - \frac{5}{6} = \frac{2}{3}$

35.  $\frac{5}{9}x - \frac{1}{2} = -\frac{7}{18}$

36.  $\frac{1}{2}x + \frac{3}{4} = \frac{1}{6}$

37.  $\frac{3}{7}x + \frac{1}{2} = 3$

38.  $-\frac{3}{4}x + \frac{5}{14} = -2$

39.  $\frac{2}{5}x = 3$

40.  $\frac{5}{8}x = -\frac{2}{3}$

41.  $x + \frac{2}{3} = \frac{3}{5}$

42.  $x - \frac{3}{4} = \frac{1}{2}$

43.  $x + \frac{2}{5} = -\frac{3}{2}$

44.  $x - \frac{3}{4} = -\frac{9}{10}$

**Answer Key.** On this part, the answer key shows the LCD used to clear the fractions. It then shows the resulting equation and the final solution.

31. LCD 24  
 $9x + 20 = 30x + 16$   
 $x = \frac{4}{21}$

32. LCD 18  
 $4x - 6 = 15x - 9$   
 $x = \frac{3}{11}$

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33. LCD 20  
 $8x + 5 = -14$   
 $x = -\frac{19}{8}$

34. LCD 30  
 $8x - 25 = 20$   
 $x = \frac{45}{8}$

35. LCD 18  
 $10x - 9 = -7$   
 $x = \frac{2}{10} = \frac{1}{5}$

36. LCD 12  
 $6x + 9 = 2$   
 $x = -\frac{7}{6}$

37. LCD 14  
 $6x + 7 = 42$   
 $x = \frac{35}{6}$

38. LCD 28  
 $-21x + 10 = -56$   
 $x = \frac{-66}{-21} = \frac{22}{7}$

39. LCD 5  
 $2x = 15$   
 $x = \frac{15}{2}$

40. LCD 24  
 $15x = -16$   
 $x = -\frac{16}{15}$

41. LCD 15  
 $15x + 10 = 9$   
 $x = -\frac{1}{15}$

42. LCD 4  
 $4x - 3 = 2$   
 $x = \frac{5}{4}$

43. LCD 10  
 $10x + 4 = -15$   
 $x = -\frac{19}{10}$

44. LCD 20  
 $20x - 15 = -18$   
 $x = -\frac{3}{20}$