Math 95
Bottoms Up Factoring
page #1

There are several different ways to factor trinomials when the leading coefficient isn't one. Your book presents two methods: trial-and-error and factoring by grouping. This worksheet will present a third method, called "bottoms up". It will be up to you to decide which method makes the most sense to you. However, most instructors use one of the book's methods: trial and error or factor by grouping (sometimes called a times c method).

This worksheet will talk about the steps for bottoms up factoring and will illustrate the steps with a specific example. Then several more examples will be presented. See if you can follow along.

To factor using bottoms up on $ax^2 + bx + c$: ALWAYS divide out any GCF before you start the steps!		Factor $2x^2 + 5x - 12$
1st	multiply a · c (1st # times last #)	$2 \cdot -12 = -24$
2nd	factor that result into all possibilities	$1 \cdot 24$ work with signs $2 \cdot 12$ later; you will $3 \cdot 8$ need one pos and $4 \cdot 6$ one neg
3rd	choose the pair that combines to the middle number, being careful with signs	choose -3 and $+8$ because $-3 \cdot 8 = -24$ AND $-3 + 8 = 5$, the middle #
4th	use those two numbers inside your parentheses (notice if you multiply the binomials at this stage, you do NOT get the original polynomial! you aren't done!)	$(x - 3)(x + 8)$
5th	now you need a "bottom" – the first number goes on the bottom of EACH number	$\left(x - \frac{3}{2}\right)\left(x + \frac{8}{2}\right)$
6th	reduce anything that reduces	$\left(x - \frac{3}{2}\right)(x + 4)$
7th	still have a bottom? go "bottoms up"! the bottom number goes right in front of x	$(2x - 3)(x + 4)$
8th	check by multiplying to see if you get the original polynomial	$(2x - 3)(x + 4)$ $2x^2 + 8x - 3x - 12$ $2x^2 + 5x - 12$ This checks!

Now look at several more examples.

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1. $6m^2 + 13m + 5$ — think $a \cdot c$ $6 \cdot 5 = 30$
factor $1 \cdot 30$, $2 \cdot 15$, $3 \cdot 10$, $5 \cdot 6$
choose choose $+3$ and $+10$ to add to $+13$ in middle and write those numbers in parentheses
$(m + 3)(m + 10)$ — doesn't multiply to original polynomial so needs a bottom – the first number 6 under EACH number

$\left(m+\frac{3}{6}\right)\left(m+\frac{10}{6}\right)$ — reduce

$\left(m+\frac{1}{2}\right)\left(m+\frac{5}{3}\right)$ — go bottoms up!

$(2m + 1)(3m + 5)$ — and check: $6m^2 + 10m + 3m + 5 \Rightarrow 6m^2 + 13m + 5$ ✓

2. $24k^2 - 19k + 2$ — $a \cdot c$ $24 \cdot 2 = 48$
factor $1 \cdot 48$, $2 \cdot 24$, $3 \cdot 16$, $4 \cdot 12$, $6 \cdot 8$
choose -3 and -16 (can you see why?)
$(k - 3)(k - 16)$ — needs a bottom (can you see why?)

$\left(k-\frac{3}{24}\right)\left(k-\frac{16}{24}\right)$ — reduce

$\left(k-\frac{1}{8}\right)\left(k-\frac{2}{3}\right)$ — time for bottoms up! (can you see why?)

$(8k - 1)(3k - 2)$ — check: $24k^2 - 16k - 3k + 2 \Rightarrow 24k^2 - 19k + 2$ ✓

3. $14y^2 - y - 3$ — $14 \cdot 3 = 42$, which can be $1 \cdot 42$ or $2 \cdot 21$ or $3 \cdot 14$ or $6 \cdot 7$
$(y + 6)(y - 7)$ — choose $+6$ and -7 (can you see why?)

$\left(y+\frac{6}{14}\right)\left(y-\frac{7}{14}\right)$ — needs a bottom (why?) and now reduce

$\left(y+\frac{3}{7}\right)\left(y-\frac{1}{2}\right)$ — not done! bottoms up!

$(7y + 3)(2y - 1)$ — check: $14y^2 - 7y + 6y - 3 \Rightarrow 14y^2 - y - 3$ ✓

4. $x^2 + 2x - 15$ — remember the "understood" 1 in front so $1 \cdot 15 = 15$, which can be $1 \cdot 15$ or $3 \cdot 5$
choose -3 and $+5$

$\left(x-\frac{3}{1}\right)\left(x+\frac{5}{1}\right)$ — remember the "understood" 1 as the bottom

$(x - 3)(x + 5)$ — reduce! Check: $x^2 + 5x - 3x - 15 \Rightarrow x^2 + 2x - 15$ ✓