

## Math 95--Factor by Grouping--page 1

This worksheet will approach factoring of trinomials of type  $ax^2 + bx + c$ , the type where the first coefficient is NOT a 1. This method relies on the technique “factoring by grouping” (sometimes called a  $\cdot c$  method).

### Part A--Factor trinomials of type $ax^2 + bx + c$ .

We’re going to factor trinomials by grouping. First, make sure the trinomial is in descending order. Then make sure all coefficients are visible--in other words, write the “understood” 1's where necessary. Here are the steps we’ll use to factor using grouping.

1st Multiply the coefficient of  $x^2$  by the constant (this is  $a \cdot c$ ).  
example:  $2a^2 + 11a + 12$  multiply  $2 \cdot 12 = 24$

2nd Factor this result into all its possibilities.

$$\begin{aligned} 24 = & 1 \cdot 24 \\ & 2 \cdot 12 \\ & 3 \cdot 8 \\ & 4 \cdot 6 \end{aligned}$$

3rd Choose the possibility that

- adds to the middle coefficient (if the original trinomial ends with an add symbol)
- subtracts to the middle coefficient (if the original trinomial ends with a subtract symbol).

So in our example, we have an ADD symbol at the end:

	<u>add to 11:</u>	
$24 = 1 \cdot 24$	$1 + 24 = 25$	
$2 \cdot 12$	$2 + 12 = 14$	
$3 \cdot 8$	$3 + 8 = 11$	This is the one!
$4 \cdot 6$	$4 + 6 = 10$	

4th Then rewrite the problem into four terms, keeping the first and last terms the same and rewriting the middle term using the two factors chosen (put the variable with these factors).

Our problem:	$2a^2 + 11a + 12$	We chose 3 and 8 ( $3 \cdot 8 = 24$ and $3 + 8 = 11$ )
	$2a^2 + 3a + 8a + 12$	OR $2a^2 + 8a + 3a + 12$

5th Finish with factoring by grouping, where you factor the first two terms, keep the middle sign, and factor the last two terms. Here’s what our example will look like.

$2a^2 + 11a + 12$	OR	$2a^2 + 11a + 12$
$2a^2 + 3a + 8a + 12$		$2a^2 + 8a + 3a + 12$
$a(2a + 3) + 4(2a + 3)$		$2a(a + 4) + 3(a + 4)$
$(2a + 3)(a + 4)$		$(a + 4)(2a + 3)$

You check by multiplying:	$(2a + 3)(a + 4)$	OR	$(a + 4)(2a + 3)$	
	$2a^2 + 8a + 3a + 12$		$2a^2 + 3a + 8a + 12$	
	$2a^2 + 11a + 12$		$2a^2 + 11a + 12$	Checks!

Several examples will help.



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So, as you are beginning to factor, you need to ALWAYS divide out the GCF first. Then continue to factor using grouping when you have  $ax^2 + bx + c$ . This next example will present how to show the GCF in all the steps. I usually prefer to just carry down the GCF in the final answer and not show it in all the other steps. On the following, I'll use an extra tabbed space to show the GCF in front of all the work.

f.  $50x^2 - 75x - 125$   
 $25(2x^2 - 3x - 5)$   
 $25(2x^2 + 2x - 5x - 5)$                       or                       $25(2x^2 - 5x + 2x - 5)$   
 $25[2x(x+1) - 5(x+1)]$                        $25[x(2x-5) + 1(2x-5)]$   
 $25(x+1)(2x-5)$                                        $25(2x-5)(x+1)$

g. If the first coefficient of the trinomial is negative, factor out a  $-1$  before you begin. It's much easier to factor if the first term is positive.

$-3x^2 - 4x + 7$   
 $-1(3x^2 + 4x - 7)$   
 $-1(3x^2 - 3x + 7x - 7)$                       or                       $-1(3x^2 + 7x - 3x - 7)$   
 $-1[3x(x-1) + 7(x-1)]$                        $-1[x(3x+7) - 1(3x+7)]$   
 $-1(x-1)(3x+7)$                                        $-1(3x+7)(x-1)$

h. Sometimes you factor out a negative GCF. Observe.

$-3x^2 - 6x + 24$   
 $-3(x^2 + 2x - 8)$   
 $-3(x+2)(x-4)$

**Factor completely.** If you don't have time to do all this worksheet, do the odds.

- |                       |                       |                       |
|-----------------------|-----------------------|-----------------------|
| 1. $2x^2 + 5x + 2$    | 2. $2x^2 + 7x + 3$    | 3. $2x^2 + 5x + 3$    |
| 4. $2x^2 + 9x + 4$    | 5. $2x^2 + 7x + 6$    | 6. $3x^2 + 8x + 5$    |
| 7. $3x^2 + 16x + 5$   | 8. $6x^2 + 7x + 1$    | 9. $6x^2 + 5x + 1$    |
| 10. $6x^2 + 11x + 5$  | 11. $6x^2 + 31x + 5$  | 12. $6x^2 + 13x + 5$  |
| 13. $6x^2 + 17x + 5$  | 14. $8x^2 + 73x + 9$  | 15. $8x^2 + 17x + 9$  |
| 16. $8x^2 + 27x + 9$  | 17. $8x^2 + 38x + 9$  | 18. $8x^2 + 22x + 9$  |
| 19. $8x^2 + 18x + 9$  | 20. $5x^2 - 16x + 3$  | 21. $5x^2 - 8x + 3$   |
| 22. $7x^2 - 29x + 4$  | 23. $7x^2 - 11x + 4$  | 24. $7x^2 - 16x + 4$  |
| 25. $3x^2 - 2x - 1$   | 26. $3x^2 - x - 2$    | 27. $3x^2 - 2x - 5$   |
| 28. $3x^2 - 14x - 5$  | 29. $10x^2 - 7x - 3$  | 30. $10x^2 - 13x - 3$ |
| 31. $10x^2 - x - 3$   | 32. $4x^2 - 4x - 15$  | 33. $4x^2 - 28x - 15$ |
| 34. $4x^2 + 11x - 15$ | 35. $4x^2 + 17x - 15$ | 36. $5x^2 + 29x - 6$  |
| 37. $5x^2 + x - 6$    | 38. $5x^2 + 13x - 6$  | 39. $5x^2 + 7x - 6$   |

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**Factor completely.** Remember the GCF first!

- |     |                         |     |                      |     |                         |
|-----|-------------------------|-----|----------------------|-----|-------------------------|
| 40. | $14x^2 + 18x + 4$       | 41. | $6x^2 + 27x + 27$    | 42. | $12x^2 + 100x + 32$     |
| 43. | $15x^2 - 50x + 15$      | 44. | $8x^2 - 36x + 40$    | 45. | $36x^2 - 102x + 30$     |
| 46. | $20x^2 - 90x - 50$      | 47. | $14x^2 - 96x - 14$   | 48. | $27x^2 - 90x - 72$      |
| 49. | $33x^4 - 55x^3 - 22x^2$ | 50. | $8x^3 + 76x^2 - 40x$ | 51. | $48x^5 + 56x^4 - 40x^3$ |

**Answer Key.**

- |     |  |     |   |     |   |
|-----|--|-----|---|-----|---|
| 1.  | $(2x+1)(x+2)$                                | 2.  | $(2x+1)(x+3)$                             | 3.  | $(2x+3)(x+1)$                               |
| 4.  | $(2x+1)(x+4)$                                | 5.  | $(2x+3)(x+2)$                             | 6.  | $(3x+5)(x+1)$                               |
| 7.  | $(3x+1)(x+5)$                                | 8.  | $(6x+1)(x+1)$                             | 9.  | $(3x+1)(2x+1)$                              |
| 10. | $(6x+5)(x+1)$                                | 11. | $(6x+1)(x+5)$                             | 12. | $(3x+5)(2x+1)$                              |
| 13. | $(3x+1)(2x+5)$                               | 14. | $(8x+1)(x+9)$                             | 15. | $(8x+9)(x+1)$                               |
| 16. | $(8x+3)(x+3)$                                | 17. | $(4x+1)(2x+9)$                            | 18. | $(4x+9)(2x+1)$                              |
| 19. | $(4x+3)(2x+3)$                               | 20. | $(5x-1)(x-3)$                             | 21. | $(5x-3)(x-1)$                               |
| 22. | $(7x-1)(x-4)$                                | 23. | $(7x-4)(x-1)$                             | 24. | $(7x-2)(x-2)$                               |
| 25. | $(3x+1)(x-1)$                                | 26. | $(3x+2)(x-1)$                             | 27. | $(3x-5)(x+1)$                               |
| 28. | $(3x+1)(x-5)$                                | 29. | $(10x+3)(x-1)$                            | 30. | $(5x+1)(2x-3)$                              |
| 31. | $(5x-3)(2x+1)$                               | 32. | $(2x+3)(2x-5)$                            | 33. | $(2x-15)(2x+1)$                             |
| 34. | $(4x+15)(x-1)$                               | 35. | $(4x-3)(x+5)$                             | 36. | $(5x-1)(x+6)$                               |
| 37. | $(5x+6)(x-1)$                                | 38. | $(5x-2)(x+3)$                             | 39. | $(5x-3)(x+2)$                               |
| 40. | $2(7x^2 + 9x + 2)$<br>$2(7x+2)(x+1)$         | 41. | $3(2x^2 + 9x + 9)$<br>$3(2x+3)(x+3)$      | 42. | $4(3x^2 + 25x + 8)$<br>$4(3x+1)(x+8)$       |
| 43. | $5(3x^2 - 10x + 3)$<br>$5(3x-1)(x-3)$        | 44. | $4(2x^2 - 9x + 10)$<br>$4(2x-5)(x-2)$     | 45. | $6(6x^2 - 17x + 5)$<br>$6(3x-1)(2x-5)$      |
| 46. | $10(2x^2 - 9x - 5)$<br>$10(2x+1)(x-5)$       | 47. | $2(7x^2 - 48x - 7)$<br>$2(7x+1)(x-7)$     | 48. | $9(3x^2 - 10x - 8)$<br>$9(3x+2)(x-4)$       |
| 49. | $11x^2(3x^2 - 5x - 2)$<br>$11x^2(3x+1)(x-2)$ | 50. | $4x(2x^2 + 19x - 10)$<br>$4x(2x-1)(x+10)$ | 51. | $8x^3(6x^2 + 7x - 5)$<br>$8x^3(3x+5)(2x-1)$ |