Math 96–Completing the Square–page #1

Completing the Square is a technique to manipulate any trinomial to the form $(x + a)^2 = b$ so you can then square root both sides and solve for the variable. Read the steps for completing the square.

1. If there's a number in front of x^2, divide by that number. You must have $1x^2$ … to complete the square.
2. Move the constant to the right side of the equals symbol.
3. Multiply the middle coefficient by ½, square the result, and add that number to both sides of the equation.
4. Factor the left side until you get (binomial)^2 and simplify the right side.
5. Square root both sides; use $\pm\sqrt{\ }$ on the constant side.
6. Simplify the radical on the constant side.
7. Isolate the variable.

Several examples follow.

a. $x^2 + 20x + 4 = 0$

$x^2 + 20x = -4$

½ (20) = 10;

$(10)^2 = 100$

$x^2 + 20x + 100 = -4 + 100$

$(x + 10)(x + 10) = 96$

$(x + 10)^2 = 96$ Now square root both sides

$x + 10 = \pm\sqrt{96}$

$x + 10 = \pm 4\sqrt{6}$

$x = -10 \pm 4\sqrt{6}$

b. $x^2 - 18x - 9 = 0$

$x^2 - 18x = 9$

½ (-18) = -9;

$(-9)^2 = 81$

$x^2 - 18x + 81 = 9 + 81$

$(x - 9)(x - 9) = 90$

$(x - 9)^2 = 90$

$x - 9 = \pm\sqrt{90}$

$x - 9 = \pm 3\sqrt{10}$

$x = 9 \pm 3\sqrt{10}$

c. $2x^2 + 20x - 6 = 0$ Divide by 2

$x^2 + 10x - 3 = 0$

$x^2 + 10x = 3$

½ (10) = 5

$(5)^2 = 25$

$x^2 + 10x + 25 = 3 + 25$

$(x + 5)(x + 5) = 28$

$(x + 5)^2 = 28$ Now square root both sides

$x + 5 = \pm 2\sqrt{7}$

$x = -5 \pm 2\sqrt{7}$

d. $3x^2 - 21x + 6 = 0$

$x^2 - 7x + 2 = 0$

$x^2 - 7x = -2$

$\frac{1}{2}\left(-\frac{7}{1}\right) = -\frac{7}{2}$

$\left(-\frac{7}{2}\right)^2 = \frac{49}{4}$

$x^2 - 7x + \frac{49}{4} = -2 + \frac{49}{4}$

$\left(x - \frac{7}{2}\right)\left(x - \frac{7}{2}\right) = -\frac{8}{4} + \frac{49}{4}$

$\left(x - \frac{7}{2}\right)^2 = \frac{41}{4}$

$x - \frac{7}{2} = \pm\sqrt{\frac{41}{4}}$

$x - \frac{7}{2} = \pm\frac{\sqrt{41}}{2}$

$x = \frac{7}{2} \pm \frac{\sqrt{41}}{2} \Rightarrow \frac{7 \pm \sqrt{41}}{2}$

e. $7x^2 + 14x + 77 = 0$

$x^2 + 2x + 11 = 0$

$x^2 + 2x = -11$

$\frac{1}{2}(2) = 1;$

$(1)^2 = 1$

$x^2 + 2x + 1 = -11 + 1$

$(x + 1)(x + 1) = -10$

$(x + 1)^2 = -10$

$x + 1 = \pm\sqrt{-10}$

$x + 1 = \pm i\sqrt{10}$

$x = -1 \pm i\sqrt{10}$

f. $8x^2 + 40x - 112 = 0$

$x^2 + 5x - 14 = 0$

$x^2 + 5x = 14$

$\frac{1}{2}(5) = \frac{5}{2}$

$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$x^2 + 5x + \frac{25}{4} = 14 + \frac{25}{4}$

$\left(x + \frac{5}{2}\right)\left(x + \frac{5}{2}\right) = \frac{56}{4} + \frac{25}{4}$

$\left(x + \frac{5}{2}\right)^2 = \frac{81}{4}$

$x + \frac{5}{2} = \pm\sqrt{\frac{81}{4}}$

$x + \frac{5}{2} = \pm\frac{9}{2}$

$x = -\frac{5}{2} \pm \frac{9}{2}$

$x = -\frac{5}{2} + \frac{9}{2}$ *or* $x = -\frac{5}{2} - \frac{9}{2}$

$x = \frac{4}{2}$ *or* $x = -\frac{14}{2}$

$x = 2$ or $x = -7$

Note: Because the answers are rational, this equation could have been solved by factoring.

Observe: $8x^2 + 40x - 112 = 0$

$8(x^2 + 5x - 14) = 0$

$8(x^2 - 2x + 7x - 14) = 0$

$8[x(x - 2) + 7(x - 2)] = 0$

$8(x - 2)(x + 7) = 0$

$x - 2 = 0$ or $x + 7 = 0$

$x = 2$ $x = -7$

You get the same results!

g. $2x^2 - 3x + 11 = 0$

$$x^2 - \frac{3}{2}x + \frac{11}{2} = 0$$

$$x^2 - \frac{3}{2}x = -\frac{11}{2}$$

$$\frac{1}{2}\left(-\frac{3}{2}\right) = -\frac{3}{4}$$

$$\left(-\frac{3}{4}\right)^2 = \frac{9}{16}$$

$$x^2 - \frac{3}{2}x + \frac{9}{16} = -\frac{11}{2} + \frac{9}{16}$$

$$\left(x - \frac{3}{4}\right)\left(x - \frac{3}{4}\right) = -\frac{88}{16} + \frac{9}{16}$$

$$\left(x - \frac{3}{4}\right)^2 = -\frac{79}{4}$$

$$x - \frac{3}{4} = \pm\sqrt{-\frac{79}{16}}$$

$$x - \frac{3}{4} = \pm\frac{i\sqrt{79}}{4}$$

$$x = \frac{3}{4} \pm \frac{i\sqrt{79}}{4} \Rightarrow \frac{3 \pm i\sqrt{79}}{4}$$

Solve these by completing the square.

1. $x^2 + 8x - 24 = 0$
2. $y^2 - 10y + 4 = 0$
3. $m^2 + 12m = 8$
4. $k^2 - 14k = -4$
5. $2z^2 + 12z + 6 = 0$
6. $3a^2 - 18a = 243$
7. $x^2 + 9x + 4 = 0$
8. $y^2 - 5y - 1 = 0$
9. $2m^2 + 14m + 8 = 0$
10. $3k^2 - 9k - 6 = 0$
11. $2z^2 + 8z + 6 = 0$
12. $4a^2 - 12a + 20 = 0$
13. $5x^2 + 2x - 4 = 0$
14. $6y^2 - 11y + 2 = 0$
15. $3m^2 + 4m = 5$
16. $2k^2 - 7k + 1 = 0$
17. $5z^2 - 6z = 8$
18. $4a^2 + 7a + 5 = 0$

Answer Key. On this answer key, I'll give a few helpful hints and then show the solutions.

1. move 24; add 16
   $x = -4 \pm 2\sqrt{10}$
2. move 4; add 25
   $y = 5 \pm \sqrt{21}$

3. add 36

   m = $-6 \pm 2\sqrt{11}$

4. add 49

   k = $7 \pm 3\sqrt{5}$

5. divide by 2; move 3; add 9

   z = $-3 \pm \sqrt{6}$

6. divide by 3; add 9

   a = $3 \pm 3\sqrt{10}$

7. move 4; add $\frac{81}{4}$

   x = $\frac{-9 \pm \sqrt{65}}{2}$

8. move 1; add $\frac{25}{4}$

   y = $\frac{5 \pm \sqrt{29}}{2}$

9. divide by 2; move 4; add $\frac{49}{4}$

   m = $\frac{-7 \pm \sqrt{33}}{2}$

10. divide by 3; move 2; add $\frac{9}{4}$

    k = $\frac{3 \pm \sqrt{17}}{2}$

11. divide by 2; move 3; add 4

    z = -2 ± 1

    z = -1 or -3

12. divide by 4; move 5; add $\frac{9}{4}$

    a = $\frac{3 \pm i\sqrt{11}}{2}$

13. divide by 5; move $\frac{4}{5}$; add $\frac{1}{25}$

    x = $\frac{-1 \pm \sqrt{21}}{5}$

14. divide by 6; move $\frac{1}{3}$; add $\frac{121}{144}$

    y = $\frac{11 \pm \sqrt{73}}{12}$

15. divide by 3; add $\frac{4}{9}$

    m = $\frac{-2 \pm \sqrt{19}}{3}$

16. divide by 2; move $\frac{1}{2}$; add $\frac{49}{16}$

    k = $\frac{7 \pm \sqrt{41}}{4}$

17. divide by 5; add $\frac{9}{25}$

    z = $\frac{3}{5} \pm \frac{7}{5}$

    z = 2 or $-\frac{4}{5}$

18. divide by 4; move $\frac{5}{4}$; add $\frac{49}{64}$

    a = $\frac{-7 \pm i\sqrt{31}}{8}$

or by factoring:

$5z^2 - 6z - 8 = 0$

$5z^2 - 10z + 4z - 8 = 0$

$5z(z - 2) + 4(z - 2) = 0$

$(z - 2)(5z + 4) = 0$ and solve:

$\left\{2, -\frac{4}{5}\right\}$ which is the same result!