Math 96– Domain and Function Notation– page #1

In Math 96, one idea we will be focused on concerns points (x, y). In points (x, y), the domain refers to the x values. The range refers to the y values. You will consider the domain and range again later in the course.

a. State the domain and range of the points (1, 3), (4, 9), (0, 1), (−5, −9).

The domain = {1, 4, 0, −5}. Each x value is listed in the set of numbers for the domain.

The range = {3, 9, 1, −9}. Each y value is listed in the set of numbers for the range.

Homework. State the domain and range of the points.

1. (2, −2), (5, −11), (−1, 7), (−3, 13)
2. (1, 1), (0, 0), (3, 9), (−5, 25)

You will also be looking at a lot of equations that have two variables (such as, $y = 2x + 3$ or $y = x^2 + 4$). One of the main ideas is to ask about the domain of the equation, in a more abstract way.

What is the domain, when you just have the equation and not specific points? The **domain** is asking about the values the x variable **can** be, without stating each individual x value. To answer that question, it's sometimes helpful to know what x cannot be so you can then state what x CAN be. Ask: Can all numbers be used for x (in other words, will any number you choose for x yield a y answer)? Are there some numbers for x that cannot be used because you cannot get a y answer? In general, the default domain is all reals unless there is some limitation on what x can be.

	Equation	**Type of Equation**	**Domain (words or interval notation)**
b.	$y = mx + b$	linear (first degree)	D: all reals or $(-\infty, \infty)$
c.	$y = x^2 + b$	parabola quadratic (second degree)	D: all reals or $(-\infty, \infty)$
d.	$y = \lvert x \rvert + b$	absolute value	D: all reals or $(-\infty, \infty)$
e.	$y = \dfrac{1}{x+b}$	rational (fractional)	

You know denominators cannot equal zero so you can begin by asking, "What can x NOT be?" You know the denominator ≠ zero, and you can find what x cannot equal. From there, the domain is what x CAN equal. Examples will help.

By the way, all these general equations use $x + b$; this encompasses using $x - b$ also. The examples will make this clear. To figure out the domain, decide if each problem looks like a, b, c, d, or e above. Observe.

f. $y = \lvert x \rvert + 3$

This equation looks like #d, absolute value. What can x be? Anything! You can use ANY number for x (positive, negative, zero, decimal, fraction) and you will be able to calculate an answer for y. Therefore, the domain (what x can be) is **all reals or $(-\infty, \infty)$**

g. $y = x^2 - 5$

This equation looks like #c, parabola. What can x be? Anything! You can use ANY number for x (positive, negative, zero, decimal, fraction) and you will be able to calculate an answer for y. Therefore, the domain (what x can be) is **all reals or $(-\infty, \infty)$**

h. $y = -3x - 7$

This equation looks like #b, linear. What can x be? Anything! You can use ANY number for x (positive, negative, zero, decimal, fraction) and you will be able to calculate an answer for y. Therefore, the domain (what x can be) is **all reals or $(-\infty, \infty)$**

i. $y = \dfrac{1}{x+3}$

This equation looks like #e, rational. What can x be? Since you have a denominator, first ask what x cannot be.

Think: denominator $\neq 0$

$x + 3 \neq 0$

$x \neq -3$

On a number line for x, you would have shading everywhere with an empty circle at $x = -3$:

Now you can state the domain for what x CAN be:

D: all reals except $x = -3$ or D: $(-\infty, -3) \cup (-3, \infty)$

What does this domain mean? It means that when you try to use -3 for x, you cannot get an answer for y. Observe.

$y = \dfrac{1}{-3+3} \Rightarrow \dfrac{1}{0}$

Since $\dfrac{1}{0}$ is undefined, you cannot get an answer for y. This means x cannot be negative 3 because you cannot get an answer for y. This is why the domain is **D: all reals except $x = -3$ or D: $(-\infty, -3) \cup (-3, \infty)$**

j. $y = \dfrac{1}{x-4}$

This equation looks like #e, rational.

Think: $x - 4 \neq 0$

$x \neq 4$

This means x cannot be 4 because you cannot get an answer for y.

Now you can state the domain for what x CAN be:

D: all reals except $x = 4$ or D: $(-\infty, 4) \cup (4, \infty)$

k. $y = \dfrac{1}{x}$

This equation looks like #e, rational.

Think: $x \neq 0$

Now state the domain for what x CAN be.

D: all reals except $x = 0$ or D: $(-\infty, 0) \cup (0, \infty)$

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l. y = $\frac{2}{3}x + 1$

This equation looks like #b, linear. What can x be? Anything! x can be any number; you will get an answer for y no matter what x value you use. Notice this is NOT a rational equation because the variable x is NOT in the denominator. The fraction $\frac{2}{3}$ is the coefficient. Another way to write this equation is y $\frac{2x}{3} + 1$.

Since x can be anything. the domain is **all reals or $(-\infty, \infty)$**

Homework. For each equation,
a) state the type of equation;
b) state the domain.

3. y = $3x - 4$

4. y = $|x| - 3$

5. y = $\frac{1}{5}x$

6. y = $\frac{1}{x+5}$

7. y = $2x^2 - 3$

8. y = $|x| + 7$

9. y = $\frac{1}{x-9}$

10. y = $x^2 + 1$

Sometimes when you talk about domain in conjunction with denominators, you have more to think about.

m. y = $\frac{1}{4x+12}$

denominator $\neq 0$
$4x + 12 \neq 0$
$4x \neq -12$
$x \neq -3$

D: all reals except x = -3 or
D: $(-\infty, -3) \cup (-3, \infty)$

n. y = $\frac{8}{5x-35}$

denominator $\neq 0$
$5x - 35 \neq 0$
$5x \neq 35$
$x \neq 7$

D: all reals except x = 7 or
D: $(-\infty, 7) \cup (7, \infty)$

o. y = $\frac{9}{x^2 + x - 30}$

denominator $\neq 0$
$x^2 + x - 30 \neq 0$
$(x - 5)(x + 6) \neq 0$
$x - 5 \neq 0$ or $x + 6 \neq 0$
$x \neq 5$ or $x \neq -6$

D: all reals except x = -6 or x = 5 or
D: $(-\infty, -6) \cup (-6, 5) \cup (5, \infty)$

Homework. State the domain.

11. $y = \dfrac{1}{7x+56}$

12. $y = \dfrac{5}{2x-30}$

13. $y = \dfrac{7}{x^2+11x+18}$

14. $y = \dfrac{3}{x^2-2x-48}$

This course will also make use of **function notation**. Function notation is an alternative way of writing equations in two variables. For example, the equation $y = 2x + 7$ can also be written in function notation: $f(x) = 2x + 7$. This is read out loud as "f of x equals 2x + 7" or "the function of x equals 2x + 7". Function notation is valuable for many reasons; we will start out using the following.

p.		
$y = 3x + 2$	could be written	$f(x) = 3x + 2$
find y when $x = 5$		$f(5)$
find y when $x = 1$		$f(1)$
find y when $x = -2$		$f(-2)$

On the left above is the first way we learned about equations. Notice it takes a lot of writing to ask to find y when you are given an x value. Function notation is a more concise way of writing the same idea. For example, on the right above, you have
$f(5)$ which means find y when $x = 5$
$f(1)$ which means find y when $x = 1$
$f(-2)$ which means find y when $x = -2$

Now observe how to go about using function notation and following those instructions. Essentially, you are given an x value which you plug into the given equation/function. Follow order of operations to find a result.

$f(x) = 3x + 2$	$f(x) = 3x + 2$	$f(x) = 3x + 2$
$f(5) = 3(\mathbf{5}) + 2$	$f(1) = 3(\mathbf{1}) + 2$	$f(-2) = 3(-\mathbf{2}) + 2$
$f(5) = 15 + 2$	$f(1) = 3 + 2$	$f(-2) = -6 + 2$
$f(5) = 17$	$f(1) = 5$	$f(-2) = -4$

The meaning of what you just calculated:

$f(5) = 17$	$f(1) = 5$	$f(-2) = -4$
$x = 5, y = 17$	$x = 1, y = 5$	$x = -2, y = -4$
point (5, 17)	point (1, 5)	point (−2, −4)

q. Given $f(x) = x^2 - 5x + 3$, find
$f(4)$
$f(0)$
$f(-7)$
$f(k)$

$f(4) = (\mathbf{4})^2 - 5(\mathbf{4}) + 3$	$f(0) = (\mathbf{0})^2 - 5(\mathbf{0}) + 3$	$f(-7) = (-\mathbf{7})^2 - 5(-\mathbf{7}) + 3$
$f(4) = 16 - 20 + 3$	$f(0) = 0 - 0 + 3$	$f(-7) = 49 + 35 + 3$
$f(4) = -1$	$f(0) = 3$	$f(-7) = 87$

$f(k) = (\mathbf{k})^2 - 5(\mathbf{k}) + 3 \Rightarrow \quad f(k) = k^2 - 5k + 3$

r. Graph $f(x) = 3x + 1$

This means graph $y = 3x + 1$. So you will build a table of values, choosing your own x values and figuring out the corresponding y values.

x	f(x) or y
x	y
−2	−5
−1	−2
0	1
1	4
2	7

Plug in your chosen x values:

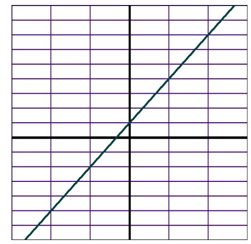
$f(-2) = 3(-2) + 1 = -6 + 1 = -5$

$f(-1) = 3(-1) + 1 = -3 + 1 = -2$

$f(0) = 3(0) + 1 = 0 + 1 = 1$

$f(1) = 3(1) + 1 = 3 + 1 = 4$

$f(2) = 3(2) + 1 = 6 + 1 = 7$



Homework.

15. $f(x) = 5x - 1$
    a. f(3)
    b. f(0)
    c. f(−4)
    d. f(k)

16. $f(x) = x^2 + 4$
    a. f(6)
    b. f(0)
    c. f(−8)
    d. f(m)

17. $f(x) = x^2 + 2x - 8$
    a. f(5)
    b. f(0)
    c. f(−3)
    d. f(z)

18. When f(3) = 10, what is the x value?

19. When f(4) = 19, what is the y value?

20. Re-write f(6) = 2 in point form.

21. Graph $f(x) = 2x + 3$

Answer Key.

1. Domain = {2, 5, −1, − 3}
   Range = {−2, −11, 7, 13}

2. Domain = {1, 0, 3, −5}
   Range = {1, 0, 9, 25}

3. a. linear
   b. all reals or $(-\infty, \infty)$

4. a. absolute value
   b. all reals or $(-\infty, \infty)$

5. a. linear
   b. all reals or $(-\infty, \infty)$

6. a. rational
   b. all reals except x = −5
   or $(-\infty, -5) \cup (-5, \infty)$

7. a. parabola
   b. all reals or $(-\infty, \infty)$

8. a. absolute value
   b. all reals or $(-\infty, \infty)$

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9.
a. rational
b. all reals except x = 9
or $(-\infty, 9) \cup (9, \infty)$

10.
a. parabola
b. all reals or $(-\infty, \infty)$

11. D: all reals except x = −8
or $(-\infty, -8) \cup (-8, \infty)$

12. D: all reals except x = 15
or $(-\infty, 15) \cup (15, \infty)$

13. $(x + 2)(x + 9) \neq 0$
$x \neq -2$ or $x \neq -9$
D: all reals except x = −9 or x = −2
or $(-\infty, -9) \cup (-9, -2) \cup (-2, \infty)$

14. $(x + 6)(x - 8) \neq 0$
$x \neq -6$ or $x \neq 8$
D: all reals except x = −6 or x = 8
or $(-\infty, -6) \cup (-6, 8) \cup (8, \infty)$

15. $f(x) = 5x - 1$
a. $f(3) = 14$
b. $f(0) = -1$
c. $f(-4) = -21$
d. $f(k) = 5k - 1$

16. $f(x) = x^2 + 4$
a. $f(6) = 40$
b. $f(0) = 4$
c. $f(-8) = 68$
d. $f(m) = m^2 + 4$

17. $f(x) = x^2 + 2x - 8$
a. $f(5) = 27$
b. $f(0) = -8$
c. $f(-3) = -5$
d. $f(z) = z^2 + 2z - 8$

18. The x value is 3.

19. The y value is 19.

20. The point is (6, 2).

21.

x	f(x) or
x	y
−2	−1
−1	1
0	3
1	5
2	7