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What Technique to Use to Solve Quadratic Equations

A. Binomials: To solve quadratic equations that are binomials:

1. Factor

a. $2x^2 + 10x = 0$
$2x(x + 5) = 0$
$2x = 0$ or $x + 5 = 0$
$x = 0$ or $x = -5$

b. $x^2 - 36 = 0$
$(x + 6)(x - 6) = 0$
$x + 6 = 0$ or $x - 6 = 0$
$x = -6$ or $x = 6$

2. Square Root Property

a. $x^2 - 12 = 0$
$x^2 = 12$
$x = \pm\sqrt{12}$
$x = \pm 2\sqrt{3}$

b. $x^2 + 45 = 0$
$x^2 = -45$
$x = \pm\sqrt{-45}$
$x = \pm 3i\sqrt{5}$

B. Trinomials: To solve quadratic equations that are trinomials:

3. Factor

a. $x^2 + 7x + 10 = 0$
$(x + 2)(x + 5) = 0$
$x + 2 = 0$ or $x + 5 = 0$
$x = -2$ or $x = -5$

b. $12x^2 + 5x - 2 = 0$
$12x^2 - 3x + 8x - 2 = 0$
$3x(4x - 1) + 2(4x - 1) = 0$
$(4x - 1)(3x + 2) = 0$
$4x - 1 = 0$ or $3x + 2 = 0$
$x = \frac{1}{4}$ or $x - \frac{2}{3}$

4. Complete the Square--best used on $1x^2 \pm$ (even)x … problems

a. $x^2 + 10x + 5 = 0$
$x^2 + 10x = -5$
$x^2 + 10x + \mathbf{25} = -5 + \mathbf{25}$
$(x + 5)^2 = 20$ now square root
$x + 5 = \pm\sqrt{20}$
$x + 5 = \pm 2\sqrt{5}$
$x = -5 \pm 2\sqrt{5}$

b. $x^2 - 16x - 8 = 0$
$x^2 - 16x = 8$
$x^2 - 16x + \mathbf{64} = 8 + \mathbf{64}$
$(x - 8)^2 = 72$
$x - 8 = \pm\sqrt{72}$
$x - 8 = \pm 6\sqrt{2}$
$x = 8 \pm 6\sqrt{2}$

5. Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a. $5k^2 - 3k + 2 = 0$ $(a = 5, b = -3, c = 2)$

$$k = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(2)}}{2(5)} = \frac{3 \pm \sqrt{9 - 40}}{10} = \frac{3 \pm \sqrt{-31}}{10} = \frac{3 \pm i\sqrt{31}}{10}$$

b. $m^2 + 6m - 4 = 0$ $(a = 1, b = 6, c = -4)$

$$m = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-4)}}{2(1)} = \frac{-6 \pm \sqrt{36 + 16}}{2} = \frac{-6 \pm \sqrt{52}}{2} = \frac{-6 \pm 2\sqrt{13}}{2}$$

$$m = \frac{2(-3 \pm \sqrt{13})}{2} = -3 \pm \sqrt{13}$$

Other Equations That Use the Quadratic Formula

Any equation that is quadratic in form can be solved using the quadratic formula. All of the following are considered quadratic in form:

$x^4 + 7x^2 + 12 = 0$	where $x^2 =$ quadratic formula
$x^4 + 9x^2 - 22 = 0$	where $x^2 =$ quadratic formula
$5x^4 - 7x^2 - 6 = 0$	where $x^2 =$ quadratic formula
$(x + 9)^2 - 8(x + 9) + 15 = 0$	where $(x + 9) =$ quadratic formula
$7(x - 5)^2 + 3(x - 5) - 4 = 0$	where $(x - 5) =$ quadratic formula

Here is one way to approach solving these type of equations.

6. $x^4 + 9x^2 + 20 = 0$
   $a = 1, b = 9, c = 20$
   Instead of saying "x = formula", we'll say "x^2 = formula". Can you figure out why?

$$x^2 = \frac{-(9) \pm \sqrt{(9)^2 - 4(1)(20)}}{2(1)} = \frac{-9 \pm \sqrt{81 - 80}}{2} = \frac{-9 \pm 1}{2}$$

$x^2 = \frac{-9+1}{2}$ $\frac{-9-1}{2}$ $\frac{-8}{2}$ sc $\frac{-10}{2}$ or so $x^2 = -4$ or -5

Now you need to square root both sides so you get x isolated:

$x^2 = -4$ or $x^2 = -5$

$x = \pm\sqrt{-4}$ or $x = \pm\sqrt{-5}$

$x = \pm 2i$ or $x = \pm i\sqrt{5}$

7. $(x + 8)^2 - 7(x + 8) - 30 = 0$
   $a = 1, b = -7, c = -30$
   Instead of saying "x = formula", we'll say "$(x + 8)$ = formula". Can you figure out why?

$$x + 8 = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-30)}}{2(1)} = \frac{7 \pm \sqrt{49 + 120}}{2} = \frac{7 \pm \sqrt{169}}{2} = \frac{7 \pm 13}{2}$$

$x + 8 = \frac{7+13}{2}$ *or* $\frac{7-13}{2}$ $\frac{20}{2}$ *or* $\frac{-6}{2}$ $8 =$ so $x + 8 = 10$ or -3

Now you need to completely isolate the variable x:

$x + 8 = 10$ or $x + 8 = -3$ (on both, subtract 8 from each side)

$x = 2$ or $x = -11$

Using Substitution on Equations Quadratic in Form

On the above equations that are quadratic in form, there is another method of approach. Substitution replaces the "unusual" term with a single variable to the first power and proceeds to solve. Observe below where #8 re-works #6 from above and #9 re-works #7 from above.

8. $x^4 + 9x^2 + 20 = 0$

Let $a = x^2$ then $a^2 = (x^2)^2 = x^4$
Then replace x^4 with a^2 and replace x^2 with a to solve:
$a^2 + 9a + 20 = 0$
$(a + 4)(a + 5) = 0$
$a + 4 = 0$ or $a + 5 = 0$
$a = -4$ or $a = -5$
Now take out a and replace with x^2 to finish solving:
$x^2 = -4$ or $x^2 = -5$
$x = \pm\sqrt{-4}$ or $x = \pm\sqrt{-5}$
$x = \pm 2i$ or $x = \pm i\sqrt{5}$ Notice you get the same answers!

9. $(x + 8)^2 - 7(x + 8) - 30 = 0$

Let $a = (x + 8)$ then $a^2 = (x + 8)^2$
Then replace $(x + 8)^2$ with a^2 and replace $(x + 8)$ with a to solve:
$a^2 - 7a - 30 = 0$
$(a + 3)(a - 10) = 0$
$a + 3 = 0$ or $a - 10 = 0$
$a = -3$ or $a = 10$
Now take out a and replace with $x + 8$ to finish solving:
$x + 8 = -3$ or $x + 8 = 10$
$x = -11$ or $x = 2$ Notice you get the same answers!

Homework. Solve by using the quadratic formula (like 6, 7) OR by substitution (like 8, 9).

1. $x^4 - 10x^2 + 24 = 0$

2. $x^4 + 2x^2 - 15 = 0$

3. $(x + 11)^2 + 7(x + 11) + 12 = 0$

4. $(x - 15)^2 - 3(x - 15) - 10 = 0$

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Answer Key. First, the solution will be shown using the quadratic formula. Then the solution will be shown using substitution.

Using quadratic formula:

1. $x^4 - 10x^2 + 24 = 0$

$a = 1,\ b = -10,\ c = 24$

$$x^2 = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(24)}}{2(1)}$$

$$x^2 = \frac{10 \pm \sqrt{100 - 96}}{2}$$

$$x^2 = \frac{10 \pm \sqrt{4}}{2}$$

$$x^2 = \frac{10 \pm 2}{2}$$

$$x^2 = \frac{12}{2} \text{ or } \frac{8}{2}$$

$x^2 = 6$ or 4

$x^2 = 6$ or $x^2 = 4$

square root both sides

$x = \pm\sqrt{6}$ or $x = \pm\sqrt{4}$

$x = \pm\sqrt{6}$ or $x = \pm 2$

2. $x^4 + 2x^2 - 15 = 0$

$a = 1,\ b = 2,\ c = -15$

$$x^2 = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-15)}}{2(1)}$$

$$x^2 = \frac{-2 \pm \sqrt{4 + 60}}{2}$$

$$x^2 = \frac{-2 \pm \sqrt{64}}{2}$$

$$x^2 = \frac{-2 \pm 8}{2}$$

$$x^2 = \frac{6}{2} \text{ or } \frac{-10}{2}$$

$x^2 = 3$ or -5

$x^2 = 3$ or $x^2 = -5$

square root both sides

$x = \pm\sqrt{3}$ or $x = \pm\sqrt{-5}$

$x = \pm\sqrt{3}$ or $x = \pm i\sqrt{5}$

3. $(x + 11)^2 + 7(x + 11) + 12 = 0$

$a = 1,\ b = 7,\ c = 12$

$$x + 11 = \frac{-(7) \pm \sqrt{(7)^2 - 4(1)(12)}}{2(1)}$$

$$x + 11 = \frac{-7 \pm \sqrt{49 - 48}}{2}$$

$$x + 11 = \frac{-7 \pm \sqrt{1}}{2}$$

$$x + 11 = \frac{-7 \pm 1}{2}$$

$$x + 11 = \frac{-6}{2} \text{ or } \frac{-8}{2}$$

$x + 11 = -3$ or -4

$x + 11 = -3$ or $x + 11 = -4$

now subtract 11 to isolate x:

$x = -14$ or $x = -15$

4. $(x - 15)^2 - 3(x - 15) - 10 = 0$

$a = 1,\ b = -3,\ c = -10$

$$x - 15 = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2}$$

$$x - 15 = \frac{3 \pm \sqrt{9 + 40}}{2}$$

$$x - 15 = \frac{3 \pm \sqrt{49}}{2}$$

$$x - 15 = \frac{3 \pm 7}{2}$$

$$x - 15 = \frac{10}{2} \text{ or } \frac{-4}{2}$$

$x - 15 = 5$ or -2

$x - 15 = 5$ or $x - 15 = -2$

now add 15 to isolate x:

$x = 20$ or $x = 13$

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Using substitution:

1. $x^4 - 10x^2 + 24 = 0$
Let $a = x^2$ so $a^2 = x^4$
$a^2 - 10a + 24 = 0$
$(a - 4)(a - 6) = 0$
$a - 4 = 0$ $a - 6 = 0$
$a = 4$ $a = 6$
Replace a with x^2
$x^2 = 4$ $x^2 = 6$
$x = \pm 2$ $x = \pm\sqrt{6}$

2. $x^4 + 2x^2 - 15 = 0$
Let $a = x^2$ so $a^2 = x^4$
$a^2 + 2x - 15 = 0$
$(a - 3)(a + 5) = 0$
$a - 3 = 0$ $a + 5 = 0$
$a = 3$ $a = -5$
Replace a with x^2
$x^2 = 3$ $x^2 = -5$
$x = \pm\sqrt{3}$ $x = \pm i\sqrt{5}$

3. $(x + 11)^2 + 7(x + 11) + 12 = 0$
Let $a = x + 11$ so $a^2 = (x + 11)^2$
$a^2 + 7a + 12 = 0$
$(a + 3)(a + 4) = 0$
$a + 3 = 0$ $a + 4 = 0$
$a = -3$ $a = -4$
Replace a with $x + 11$
$x + 11 = -3$ $x + 11 = -4$
$x = -14$ $x = -15$

4. $(x - 15)^2 - 3(x - 15) - 10 = 0$
Let $a = x - 15$ so $a^2 = (x - 15)^2$
$a^2 - 3a - 10 = 0$
$(a + 2)(a - 5) = 0$
$a + 2 = 0$ $a - 5 = 0$
$a = -2$ $a = 5$
Replace a with $x - 15$
$x - 15 = -2$ $x - 15 = 5$
$x = 13$ $x = 20$