Math 96--Quadratic Formula–page 1

A–Quadratic Formula. Use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve quadratic equations $ax^2 + bx + c = 0$ when the equations can't be factored. To use the quadratic formula, the equation must be in standard form. Although the formula says "x= ", this formula can actually be used for any quadratic equation regardless of the variable. So, if the problem is in the variable "m", the formula would say, "m= "; if the problem is in the variable "y", the formula would say, "y= ".

Remember to use the coefficients a, b, and c (along with the sign!) in the formula. Remember any "understood" 1 if there isn't a coefficient written. Also, remember that the equation must be equal to zero before you begin. I also recommend that the equation have whole numbers only. If the equation has fractions or decimals, manipulate the equation until it has whole numbers only. If the equation has fractions, multiply the entire equation by the LCD to eliminate the denominators. If the equation has decimals, multiply by 10, 100, 1000, etc. to remove the decimal point. Your results may be rational, irrational, or complex.

Solve. State what type of result this is: rational, irrational, or complex.

1. $3x^2 + 7x + 1 = 0$
2. $5m^2 + 9m + 2 = 0$
3. $4a^2 + 6a - 3 = 0$
4. $y^2 - 5y - 2 = 0$
5. $z^2 - 7z - 3 = 0$
6. $k^2 + 12k - 5 = 0$
7. $3x^2 + x + 4 = 0$
8. $9h^2 = 9h + 2$
9. $5t^2 + 6t = 7$
10. $-7x = 9 - 2x^2$
11. $0.2x^2 + 0.5x - 1.2 = 0$
12. $\frac{1}{2}x^2 - \frac{2}{3}x - \frac{5}{6} = 0$

Sometimes, you don't know a problem is quadratic until you begin simplifying. Observe.

a. $x(x + 8) - 3(x + 4) = 20$
$x^2 + 8x - 3x - 12 = 20$ quadratic--power of 2!
$x^2 + 5x - 12 - \mathbf{20} = 0$
$x^2 + 5x - 32 = 0$
$a = 1, b = 5, c = -32$; use quadratic formula! $x = \dfrac{-5 \pm \sqrt{153}}{2}$, irrational

13. Solve: $5(z - 6) + z(z + 3) = -11$

Answer Key

1. a = 3, b = 7, c = 1

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(3)(1)}}{2(3)} = \frac{-7 \pm \sqrt{49 - 12}}{6} = \frac{-7 \pm \sqrt{37}}{6}$$

irrational

2. a = 5, b = 9, c = 2

$$m = \frac{-(9) \pm \sqrt{(9)^2 - 4(5)(2)}}{2(5)} = \frac{-9 \pm \sqrt{81 - 40}}{10} = \frac{-9 \pm \sqrt{41}}{10}$$

irrational

3. a = 4, b = 6, c = -3

$$a = \frac{-(6) \pm \sqrt{(6)^2 - 4(4)(-3)}}{2(4)} = \frac{-6 \pm \sqrt{36 + 48}}{8} = \frac{-6 \pm \sqrt{84}}{8} = \frac{-6 \pm 2\sqrt{21}}{8} = \frac{-3 \pm \sqrt{21}}{4}$$

irrational

4. a = 1, b = -5, c = -2

$$y = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-2)}}{2(1)} = \frac{5 \pm \sqrt{25 + 8}}{2} = \frac{5 \pm \sqrt{33}}{2}$$

irrational

5. a = 1, b = -7, c = -3

$$z = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-3)}}{2(1)} = \frac{7 \pm \sqrt{49 + 12}}{2} = \frac{7 \pm \sqrt{61}}{2}$$

irrational

6. a = 1, b = 12, c = -5

$$k = \frac{-(12) \pm \sqrt{(12)^2 - 4(1)(-5)}}{2(1)} = \frac{-12 \pm \sqrt{144 + 20}}{2} = \frac{-12 \pm \sqrt{164}}{2} = \frac{-12 \pm 2\sqrt{41}}{2} = -6 \pm \sqrt{41}$$

irrational

7. a = 3, b = 1, c = 4

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(3)(4)}}{2(3)} = \frac{-1 \pm \sqrt{1 - 48}}{6} = \frac{-1 \pm \sqrt{-47}}{6} \quad \text{so } x = \frac{-1 \pm i\sqrt{47}}{6}$$

complex

8. a = 9, b = −9, c = −2 (you moved 9h and 2 across = so signs changed)

$$h = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(9)(-2)}}{2(9)} = \frac{9 \pm \sqrt{81+72}}{18} = \frac{9 \pm \sqrt{153}}{18} = \frac{9 \pm 3\sqrt{17}}{18} = \frac{3 \pm \sqrt{17}}{6}$$

irrational

9. a = 5, b = 6, c = −7 (you moved 7 across = so sign changed)

$$t = \frac{-(6) \pm \sqrt{(6)^2 - 4(5)(-7)}}{2(5)} = \frac{-6 \pm \sqrt{36+140}}{10} = \frac{-6 \pm \sqrt{176}}{10} = \frac{-6 \pm 4\sqrt{11}}{10} = \frac{-3 \pm 2\sqrt{11}}{5}$$

irrational

10. a = 2, b = −7, c= −9 (you moved 9 and $2x^2$ across = so signs changed)

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-9)}}{2(2)} = \frac{7 \pm \sqrt{49+72}}{4} = \frac{7 \pm \sqrt{121}}{4} = \frac{7 \pm 11}{4}$$

You cannot stop--no radicals left so you need to finish! Now separate this result into two problems:

$$x = \frac{7+11}{4} = \frac{18}{4} = \frac{9}{2} \qquad \text{or} \qquad x = \frac{7-11}{4} = \frac{-4}{4} = -1$$

rational

Remember, if you get "normal" answers, then you could have factored instead of using the quadratic formula. Let's see what happens when we factor this problem instead.

Original problem:	$2x^2 - 7x - 9 = 0$		
Factor:	$2x^2 + 2x - 9x - 9 = 0$		
	$2x(x + 1) - 9(x + 1) = 0$		
	$(x + 1)(2x - 9) = 0$		
Set to zero:	$x + 1 = 0$	or	$2x - 9 = 0$
Solve:	$x = -1$	or	$2x = 9$
	$x = -1$	or	$x = \frac{9}{2}$

The solution is still x = −1 or $x = \frac{9}{2}$! You get the same answers either way you work this problem. Cool!☺!

11. $0.2x^2 + 0.5x - 1.2 = 0$ original problem
$2x^2 + 5x - 12 = 0$ multiply by 10 to get whole numbers
a = 2, b = 5, c = −12 identify a, b, and c

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(2)(-12)}}{2(2)} = \frac{-5 \pm \sqrt{25+96}}{4} = \frac{-5 \pm \sqrt{121}}{4} = \frac{-5 \pm 11}{4}$$

$$x = \frac{-5+11}{4} \text{ or } \frac{-5-11}{4} \text{ so } x = \frac{6}{4} \text{ or } \frac{-16}{4} \text{ so } x = \frac{3}{2} \text{ or } -4$$

rational

12. $\frac{1}{2}x^2 - \frac{2}{3}x - \frac{5}{6} = 0$. original problem

$3x^2 - 4x - 5 = 0$ multiply by LCD 6 to get whole numbers

$a = 3,\ b = -4,\ c = -5$ identify a, b, and c

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-5)}}{2(3)} = \frac{4 \pm \sqrt{16+60}}{6} = \frac{4 \pm \sqrt{76}}{6} = \frac{4 \pm 2\sqrt{19}}{6} = \frac{2 \pm \sqrt{19}}{3}$$

irrational

13. $5(z - 6) + z(z + 3) = -11$

$5z - 30 + z^2 + 3z = -11$

$z^2 + 8z - 30 = -11$

$z^2 + 8z - 30 \mathbf{+ 11} = 0$

$z^2 + 8z - 19 = 0$

$a = 1,\ b = 8,\ c = -19$

$$z = \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(-19)}}{2(1)} = \frac{-8 \pm \sqrt{64+76}}{2} = \frac{-8 \pm \sqrt{140}}{2} = \frac{-8 \pm 2\sqrt{35}}{2} = -4 \pm \sqrt{35}$$

irrational

B–Quadratic Word Problems. Some quadratic word problems use the formulas for area. To finish a word problem when you use the quadratic formula, you will need a calculator to find the result to the square root. Observe the example: The length of a rectangle is 1 inch more than 3 times the width. The area is 14 square inches. Find the length and width.

$x =$ width length (width) = area

$3x + 1 =$ length $(3x + 1)(x) = 14$

$3x^2 + x = 14$

$3x^2 + x - 14 = 0$

At this point, you can choose to see if this trinomial factors or you can immediately use the quadratic formula to find results for x. To use the quadratic formula, $a = 3,\ b = 1,\ c = -14$.

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(3)(-14)}}{2(3)} = \frac{-1 \pm \sqrt{1+168}}{6} = \frac{-1 \pm \sqrt{169}}{6} = \frac{-1 \pm 13}{6} \text{ so } x = \frac{12}{6} \text{ or } -\frac{14}{6}$$

$x = 2$ or $-\frac{7}{3}$ (which we won't use in this word problem–why?)

x = width = 2 feet

3x + 1 = length = 3(2) + 1 = 7 feet

Set up and solve.

14. The length of a rectangle is 2 feet less than 6 times the width. The area is 48 square feet. Find the length and width.

15. The length of a rectangle is 1 foot more than 3 times the width. The area is 80 square feet. Find the length and width.

Answer Key. I'll show the labels, the original equation, the equation equal to zero, and the results. All solving is left up to you!

14. x = width = **3 ft**
$6x - 2$ = length = **16 ft**
$(6x - 2)(x) = 48$
$6x^2 - 2x - 48 = 0$

15. x = width = **5 ft**
$3x + 1$ = length = **16 ft**
$(3x + 1)(x) = 80$
$3x^2 + x - 80 = 0$

C–Develop the quadratic formula. Where did the quadratic formula come from? It is actually developed using completing the square. Observe the steps. See if you can follow what occurs.

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Think: $\left(\frac{1}{2}\right)\left(\frac{b}{a}\right) = \frac{b}{2a}$; then think $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)\left(x + \frac{b}{2a}\right) = -\frac{c}{a} \cdot \frac{4a}{4a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$$

Think: $\frac{-4ac}{4a^2} + \frac{b^2}{4a^2} \Rightarrow \frac{-4ac + b^2}{4a^2} \Rightarrow \frac{b^2 - 4ac}{4a^2}$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Now square root both sides

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

Think: $\pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \Rightarrow \pm\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}} \Rightarrow \pm\frac{\sqrt{b^2 - 4ac}}{2a}$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quadratic formula!