Math 96--Radicals #3--Rationalize Denominators--page 1

Background–Before we start to talk about rationalizing denominators, we need to review conjugates. Recall, conjugates are binomials with identical terms and opposite middle signs. Observe.

Binomial	Conjugate
$x + 8$	$x - 8$
$m - 7$	$m + 7$
$2k + 5$	$2k - 5$
$9m - 4$	$9m + 4$

What you really need to know about conjugates is what happens when you multiply them together. Observe.

$(a + b)(a - b)$
$a^2 - ab + ab - b^2$
$a^2 - b^2$

Every time you multiply with conjugates, you get the difference of squares! This allows you to multiply conjugates quickly. You can multiply the first terms and last terms and just skip multiplying the outer/inner because they're going to be identical with opposite signs so will cancel out anyway. Observe the fast multiplication.

Binomial	Conjugate	Product (First/Last)
$x + 8$	$x - 8$	$x^2 - 64$
$m - 7$	$m + 7$	$m^2 - 49$
$2k + 5$	$2k - 5$	$4k^2 - 25$
$9m - 4$	$9m + 4$	$81m^2 - 16$

Where this benefits us is with conjugates that contain radicals. Observe.

Binomial	Conjugate	Product (First/Last)
$8 + \sqrt{3}$	$8 - \sqrt{3}$	$64 - 3 = 61$
$9 - \sqrt{5}$	$9 + \sqrt{5}$	$81 - 5 = 76$
$\sqrt{11} + 2$	$\sqrt{11} - 2$	$11 - 4 = 7$
$\sqrt{5} - 7$	$\sqrt{5} + 7$	$5 - 49 = -44$

Homework. Write the conjugate and find the product.

	Binomial	Conjugate	Product (First/Last)
1.	$6 + \sqrt{3}$		
2.	$8 - \sqrt{6}$		
3.	$\sqrt{17} + 3$		
4.	$\sqrt{2} - 10$		

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Part A--Rationalize Radicals with Binomial Denominators. One of the rules of radicals says that you cannot leave a radical in the denominator. You have to multiply the fraction by a form of one to turn the denominator into a rational number. Observe the process to rationalize when there are binomials in the denominator. You will multiply by the conjugate of the denominator, in the numerator and in the denominator to keep the fraction balanced (this is your form of one). After you have done all the multiplication, you may need to reduce. I recommend you don't distribute in the numerator so you can reduce. Then you can distribute in the numerator later. If you distribute in the numerator immediately, then you'll need to factor the numerator later to reduce the fraction.

a. $\dfrac{3}{5+\sqrt{7}}$

$$\left(\frac{3}{5+\sqrt{7}}\right)\cdot\left(\frac{5-\sqrt{7}}{5-\sqrt{7}}\right)=\frac{3\left(5-\sqrt{7}\right)}{25-7}=\frac{3\left(5-\sqrt{7}\right)}{18}=\frac{5-\sqrt{7}}{6}$$

b. $\dfrac{8}{2-\sqrt{5}}$

$$\left(\frac{8}{2-\sqrt{5}}\right)\cdot\left(\frac{2+\sqrt{5}}{2+\sqrt{5}}\right)=\frac{8\left(2+\sqrt{5}\right)}{4-5}=\frac{8\left(2+\sqrt{5}\right)}{-1}=-8\left(2+\sqrt{5}\right)\ or\ -16-8\sqrt{5}$$

c. $\dfrac{6}{\sqrt{2}-4}$

$$\left(\frac{6}{\sqrt{2}-4}\right)\cdot\left(\frac{\sqrt{2}+4}{\sqrt{2}+4}\right)=\frac{6\left(\sqrt{2}+4\right)}{2-16}=\frac{6\left(\sqrt{2}+4\right)}{-14}=-\frac{6\left(\sqrt{2}+4\right)}{14}=-\frac{3\left(\sqrt{2}+4\right)}{7}\ or\ -\frac{3\sqrt{2}+12}{7}$$

d. $\dfrac{5}{\sqrt{11}+3}$

$$\left(\frac{5}{\sqrt{11}+3}\right)\cdot\left(\frac{\sqrt{11}-3}{\sqrt{11}-3}\right)=\frac{5\left(\sqrt{11}-3\right)}{11-9}=\frac{5\left(\sqrt{11}-3\right)}{2}\ or\ \frac{5\sqrt{11}-15}{2}$$

HOMEWORK. Rationalize the following.

5. $\dfrac{8}{7+\sqrt{3}}$

6. $\dfrac{11}{3-\sqrt{5}}$

7. $\dfrac{10}{\sqrt{11}+3}$

8. $\dfrac{4}{\sqrt{5}-7}$

9. $\dfrac{3}{\sqrt{6}+\sqrt{2}}$

10. $\dfrac{7}{\sqrt{10}-\sqrt{3}}$

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Part B--Rationalize Radicals with Monomial Denominators. You cannot leave a radical in the denominator; you cannot leave a denominator under the radical. You must rationalize to turn the denominator into a whole number. If you have a one term (monomial) denominator, you multiply by single radicals to make a "perfect"; you may need to factor the radicand into primes first. The index tells you how many factors that you need to make a "perfect". You may then still need to simplify, according to radical rules and/or according to fraction rules. There are a couple of different approaches to this type of rationalizing; observe the following.

e. $\frac{5}{\sqrt{3}}$

$$\frac{5}{\sqrt{3}}=\frac{5}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{5\sqrt{3}}{\sqrt{3\cdot 3}}=\frac{5\sqrt{3}}{3}$$

f. $\frac{\sqrt{5}}{\sqrt{3}}$

$$\frac{\sqrt{5}}{\sqrt{3}}=\frac{\sqrt{5}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{15}}{\sqrt{3\cdot 3}}=\frac{\sqrt{15}}{3}$$

g. $\frac{7}{\sqrt{12}}$

$$\frac{7}{\sqrt{12}}=\frac{7}{\sqrt{12}}\cdot\frac{\sqrt{12}}{\sqrt{12}}=\frac{7\sqrt{12}}{\sqrt{12\cdot 12}}=\frac{7\sqrt{12}}{12}=\frac{7\sqrt{2\cdot 2\cdot 3}}{12}=\frac{7\cdot 2\sqrt{3}}{12}=\frac{14\sqrt{3}}{12}=\frac{7\sqrt{3}}{6}$$

OR

$$\frac{7}{\sqrt{12}}=\frac{7}{\sqrt{2\cdot 2\cdot 3}}=\frac{7}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{7\sqrt{3}}{2\cdot 3}=\frac{7\sqrt{3}}{6}$$

OR

$$\frac{7}{\sqrt{12}}=\frac{7}{\sqrt{12}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{7\sqrt{3}}{\sqrt{36}}=\frac{7\sqrt{3}}{6}$$

h. $\frac{\sqrt{3}}{\sqrt{20}}$

$$\frac{\sqrt{3}}{\sqrt{20}}=\frac{\sqrt{3}}{\sqrt{20}}\cdot\frac{\sqrt{20}}{\sqrt{20}}=\frac{\sqrt{60}}{20}=\frac{\sqrt{2\cdot 2\cdot 3\cdot 5}}{20}=\frac{2\sqrt{15}}{20}=\frac{\sqrt{15}}{10}$$

OR

$$\frac{\sqrt{3}}{\sqrt{20}}=\frac{\sqrt{3}}{\sqrt{2\cdot 2\cdot 5}}=\frac{\sqrt{3}}{2\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{\sqrt{15}}{2\cdot 5}=\frac{\sqrt{15}}{10}$$

i. $\frac{12}{\sqrt{75}}$

$$\frac{12}{\sqrt{75}}=\frac{12}{\sqrt{75}}\cdot\frac{\sqrt{75}}{\sqrt{75}}=\frac{12\sqrt{75}}{75}=\frac{12\sqrt{3\cdot 5\cdot 5}}{75}=\frac{12\cdot 5\sqrt{3}}{75}=\frac{60\sqrt{3}}{75}=\frac{4\sqrt{3}}{5}$$

OR

$$\frac{12}{\sqrt{75}}=\frac{12}{\sqrt{3\cdot 5\cdot 5}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{12\sqrt{3}}{\sqrt{3\cdot 3\cdot 5\cdot 5}}=\frac{12\sqrt{3}}{3\cdot 5}=\frac{12\sqrt{3}}{15}=\frac{4\sqrt{3}}{5}$$

HOMEWORK. Rationalize the denominators.

11. $\frac{\sqrt{7}}{\sqrt{2}}$ 12. $\frac{\sqrt{3}}{\sqrt{8}}$ 13. $\frac{\sqrt{7}}{\sqrt{12}}$ 14. $\frac{\sqrt{11}}{\sqrt{20}}$

15. $\frac{18}{\sqrt{24}}$ 16. $\frac{4}{\sqrt{18}}$ 17. $\frac{6}{\sqrt{40}}$ 18. $\frac{10}{\sqrt{12}}$

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Part C–Fractions Under Radicals and Rationalizing–Sometimes, you'll have a fraction under a radical. Since all denominators should be rational, you'll need to rationalize. I would first re-write the problem as a fraction with the radical on the numerator and the radical on the denominator. Then proceed. Observe.

j. $\sqrt{\frac{2}{5}} = \frac{\sqrt{2}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$

k. $\sqrt{\frac{9}{50}} = \frac{\sqrt{9}}{\sqrt{50}} = \frac{3}{\sqrt{50}} \cdot \frac{\sqrt{50}}{\sqrt{50}} = \frac{3\sqrt{50}}{50} = \frac{3\sqrt{2 \cdot 5 \cdot 5}}{50} = \frac{15\sqrt{2}}{50} = \frac{3\sqrt{2}}{10}$

l. $\sqrt{\frac{7}{18}} = \frac{\sqrt{7}}{\sqrt{18}} \cdot \frac{\sqrt{18}}{\sqrt{18}} = \frac{\sqrt{7 \cdot 2 \cdot 3 \cdot 3}}{18} = \frac{3\sqrt{14}}{18} = \frac{\sqrt{14}}{6}$

HOMEWORK. Simplify.

19. $\sqrt{\frac{3}{7}}$ 20. $\sqrt{\frac{49}{20}}$ 21. $\sqrt{\frac{3}{10}}$ 22. $\sqrt{\frac{5}{12}}$

Part D–Combination Problems–Sometimes, you'll need to simplify a problem where you need to use several different techniques. Observe.

m. $\sqrt{12} + 7\sqrt{3} + \frac{4}{\sqrt{3}}$

$\sqrt{2 \cdot 2 \cdot 3} + 7\sqrt{3} + \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$2\sqrt{3} + 7\sqrt{3} + \frac{4\sqrt{3}}{3}$ Now get a common denominator!

$\frac{6\sqrt{3}}{3} + \frac{21\sqrt{3}}{3} + \frac{4\sqrt{3}}{3}$

$\frac{31\sqrt{3}}{3}$

Homework. Simplify.

23. $\sqrt{18} + 5\sqrt{2} + \frac{7}{\sqrt{2}}$

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Answer Key.

1. $6 - \sqrt{3}$
   $36 - 3$
   33

2. $8 + \sqrt{6}$
   $64 - 6$
   58

3. $\sqrt{17} - 3$
   $17 - 9$
   8

4. $\sqrt{2} + 10$
   $2 - 100$
   -98

5. $\frac{4(7-\sqrt{3})}{23}$ or $\frac{28-4\sqrt{3}}{23}$

6. $\frac{11(3+\sqrt{5})}{4}$ or $\frac{33+11\sqrt{5}}{4}$

7. $5(\sqrt{11}-3)$ or $5\sqrt{11}-15$

8. $\frac{4(\sqrt{5}+7)}{-44} = -\frac{4(\sqrt{5}+7)}{44} = -\frac{\sqrt{5}+7}{11}$

9. $\frac{3(\sqrt{6}-\sqrt{2})}{4}$ or $\frac{3\sqrt{6}-3\sqrt{2}}{4}$

10. $\frac{7(\sqrt{10}+\sqrt{3})}{7} = \sqrt{10}+\sqrt{3}$

11. $\frac{\sqrt{14}}{2}$

12. $\frac{\sqrt{6}}{4}$

13. $\frac{\sqrt{21}}{6}$

14. $\frac{\sqrt{55}}{10}$

15. $\frac{3\sqrt{6}}{2}$

16. $\frac{2\sqrt{2}}{3}$

17. $\frac{3\sqrt{10}}{10}$

18. $\frac{5\sqrt{3}}{3}$

19. $\frac{\sqrt{21}}{7}$

20. $\frac{7\sqrt{5}}{10}$

21. $\frac{\sqrt{30}}{10}$

22. $\frac{\sqrt{15}}{6}$

23. $\frac{23\sqrt{2}}{2}$