Math 95--Factoring Worksheet #1--page 1

This worksheet will present ideas on how to factor trinomials of type $x^2 + bx + c$, where the coefficient of x^2 is an "understood" 1.

Part A--Factoring trinomials of type $x^2 + bx + c$. Follow the following example, step-by-step.

a. Factor $x^2 + 7x + 12$

1st Write two sets of parentheses. Leave space inside to write the factors.
( )( )

2nd Factor x^2 into x times x, placing one x in the first position in each set of parentheses.
(x)(x)

3rd Decide upon the signs you'll be using.
If trinomial ends in a + symbol, use both + or both –, using the sign of the middle term.
If a trinomial ends in a – symbol, use one + and one –.

This trinomial ends in + so use sign of middle--use both + .
(x +)(x +)

4th Factor the ending number 12 into all its possibilities. Then choose the possibility that will add to the middle number if the trinomial ended in a + ; choose the possibility that will subtract to the middle number if the trinomial ended in a –.

Factors of 12	Add to the middle, using both + signs	
$1 \cdot 12$	$1 + 12 = 13$	
$2 \cdot 6$	$2 + 6 = 8$	
$3 \cdot 4$	$3 + 4 = 7$	This works!

5th Place 3 and 4 in the second position in the parentheses.

$(x + 3)(x + 4)$

6th You can check by multiplying: $(x + 3)(x + 4) = x^2 + 4x + 3x + 12 = x^2 + 7x + 12$

This process gets faster as you practice. Observe the following.

b. Factor $x^2 + 22x + 72$ Use both + since ending symbol is a + and middle is a +

$(x +)(x +)$

Factors of 72	Add to middle, using both + signs	
$1 \cdot 72$	$1 + 72 = 73$	
$2 \cdot 36$	$2 + 36 = 38$	
$3 \cdot 24$	$3 + 24 = 27$	
$4 \cdot 18$	$4 + 18 = 22$	This works!
$6 \cdot 12$	$6 + 12 = 18$	
$8 \cdot 9$	$8 + 9 = 17$	

$(x + 4)(x + 18)$ Check: $(x+ 4)(x+ 18)= x^2 + 18x + 4x + 72= x^2 + 22x + 72$

c. Factor $m^2 - 8m + 15$ Use both – since ending symbol is a + and middle is a –

$(m -)(m -)$

Factors of 15	Add to middle, using both – signs	
$-1 \cdot -15$	$-1 + (-15) = -16$	
$-3 \cdot -5$	$-3 + (-5) = -8$	This works!

$(m - 3)(m - 5)$ Check: $(m-3)(m-5)= m^2 -5m -3m + 15= m^2 -8m + 15$

Math 95--Factoring Worksheet #1--page 2

d. **Factor $a^2 + 4a - 21$** Use one pos, one neg since ending symbol is a –

$(a + \ )(a - \ )$

Factors of 21	Subtract to middle, using one pos/one neg
$1 \cdot 21$	$1 - 21 = -20$
	$21 - 1 = 20$
$3 \cdot 7$	$3 - 7 = -4$
	$7 - 3 = 4$ This works!

$(a + 7)(a - 3)$ Check: $(a+ 7)(a -3)= a^2 -3a + 7a -21 = a^2 + 4a - 21$

e. **Factor $k^2 - 10k - 24$** Use one pos, one neg since ending symbol is a –

Factors of 24	Subtract to middle, using one pos/one neg
$1 \cdot 24$	$1 - 24 = -23$
	$24 - 1 = 23$
$2 \cdot 12$	$2 - 12 = -10$ This works!
	$12 - 2 = 10$
$3 \cdot 8$	$3 - 8 = -5$
	$8 - 3 = 5$
$4 \cdot 6$	$4 - 6 = -2$
	$6 - 4 = 2$

$(k + 2)(k - 12)$ Check: $(k+ 2)(k -12)= k^2 -12k + 2k -24 = k^2 - 10k - 24$

Part B--Factoring using a GCF first. Sometimes, you need to divide a greatest common factor (GCF) out of the trinomial before you begin the process of factoring as you did above. The following examples will divide out the greatest common factor (GCF) first and then factor using the techniques for trinomials.

f. **Factor $7m^2 + 35m + 42$**

First, notice that all three terms of the trinomial divide by 7 so 7 is the GCF. Divide out 7 and show it in front of the result.

$7m^2 + 35m + 42$

$7(m^2 + 5m + 6)$ Now use both + for the factors

$7(m + \ )(m + \ )$

Factors of 6	Add to middle, using both +
$1 \cdot 6$	$1 + 6 = 7$
$2 \cdot 3$	$2 + 3 = 5$ This works!

$7(m + 2)(m + 3)$ To check, multiply the binomials first and then multiply that result by 7.

Check: $7(m + 2)(m + 3)$

$7(m^2 + 3m + 2m + 6)$

$7(m^2 + 5m + 6)$

$7m^2 + 35m + 42$

g. **Factor $2z^2 - 28z - 64$** Divide out GCF 2

$2(z^2 - 14z - 32)$ Signs are one pos/one neg

$2(z + \ )(z - \ )$

Factors of 32	Subtract to middle, using one pos/one neg
$1 \cdot 32$	$1 - 32 = -31$
$2 \cdot 16$	$2 - 16 = -14$ This works!
$4 \cdot 8$	$4 - 8 = -4$

$2(z + 2)(z - 16)$

Math 95--Factoring Worksheet #1--page 3

h. **Factor $9a^8 - 54a^7 + 72a^6$** Divide out GCF $9a^6$ (divide numbers, subtract exponents)

$9a^6(a^2 - 6a + 8)$ Use both –

$9a^6(a -)(a -)$

Factors of 8	Add to middle, using both neg	
$-1 \cdot -8$	$-1 + -8 = -9$	
$-2 \cdot -4$	$-2 + -4 = -6$	This works!

$9a^6(a - 2)(a - 4)$ To check, multiply the binomials first and then distribute the GCF:

$(a + 2)(a + 4) = a^2 + 4a + 2a + 8 = a^2 + 6a + 8$

Then $9a^6(a^2 + 6a + 8) = 9a^8 + 54a^7 + 72a^6$ and this checks

i. **Factor $x^2(x + 12) + 9x(x + 12) - 22(x + 12)$** GCF is the phrase $(x + 12)$ so divide $(x + 12)$ out

$(x + 12)(x^2 + 9x - 22)$ Use one pos/one neg

$(x + 12)(x +)(x -)$

Factors of 22	Subtract to middle, using one pos/one neg	
$1 \cdot 22$	$22 - 1 = 21$	
$2 \cdot 11$	$11 - 2 = 9$	This works!

$(x + 12)(x + 11)(x - 2)$

In fact, when you are factoring, I want you to first factor out any GCF!

Organized thoughts:	Signs of Polynomial		Signs of Factors
	+	+	(+)(+)
	-	+	(-)(-)
	+	-	(+)(-)
	-	-	(+)(-)

If the polynomial ends with a "plus" symbol, add to the middle term using both pos or using both neg.
If the polynomial ends with a "subtract" symbol, subtract to the middle term using one pos and one neg.

Make sure the polynomial is in descending order. If you see $m^2 + 9 + 10m$, rearrange it to $m^2 + 10m + 9$.

Always divide out a GCF, if there is one, and then continue to factor the trinomial.

Now you just need some practice! Factor the following. Make sure the polynomial is in descending order.

1.	$x^2 + 9x + 20$	2.	$m^2 + 4m + 4$	3.	$k^2 + 6 + 7k$
4.	$y^2 + 7y + 10$	5.	$a^2 + 12 + 7a$	6.	$z^2 - 11z + 30$
7.	$x^2 - 16x + 28$	8.	$v^2 - 12v + 27$	9.	$c^2 - 10c + 25$
10.	$w^2 - 25w + 24$	11.	$f^2 + 3f - 10$	12.	$x^2 + 2x - 15$
13.	$a^2 - 12 + a$	14.	$m^2 + 5m - 24$	15.	$y^2 + 5y - 6$
16.	$y^2 - 3y - 18$	17.	$z^2 - 7z - 18$	18.	$x^2 - 8x - 48$
19.	$y^2 - 5y - 6$	20.	$k^2 - 13k - 48$		

Factor completely. Remember the GCF first!

21.	$2x^2 + 30x + 108$	22.	$3x^2 - 27x + 54$	23.	$7x^2 - 14x - 336$

Math 95--Factoring Worksheet #1--page 4

24. $5x^8 + 35x^7 - 90x^6$

25. $11x^4y + 33x^3y - 198x^2y$

26. $x^2(a + 2) + 9x(a + 2) + 14(a + 2)$

27. $x^2(m - 3) - 5x(m - 3) + 6(m - 3)$

28. $x^2(x + 5) + 7x(x + 5) - 18(x + 5)$

Answer Key.

1. $(x+ 4)(x+ 5)$

2. $(m+ 2)(m+ 2)$

3. $(k+ 1)(k+ 6)$

4. $(y+ 2)(y+ 5)$

5. $(a+ 3)(a+ 4)$

6. $(z - 5)(z - 6)$

7. $(x - 2)(x - 14)$

8. $(v - 3)(v - 9)$

9. $(c - 5)(c - 5)$

10. $(w - 1)(w - 24)$

11. $(f + 5)(f - 2)$

12. $(x + 5)(x - 3)$

13. $(a + 4)(a - 3)$

14. $(m + 8)(m - 3)$

15. $(y + 6)(y - 1)$

16. $(y + 3)(y - 6)$

17. $(z + 2)(z - 9)$

18. $(x + 4)(x - 12)$

19. $(y + 1)(y - 6)$

20. $(k + 3)(k - 16)$

21. $2x^2 + 30x + 108$
$2(x^2 + 15x + 54)$
$2(x + 6)(x + 9)$

22. $3x^2 - 27x + 54$
$3(x^2 - 9x + 18)$
$3(x - 3)(x - 6)$

23. $7x^2 - 14x - 336$
$7(x^2 - 2x - 48)$
$7(x + 6)(x - 8)$

24. $5x^8 + 35x^7 - 90x^6$
$5x^6(x^2 + 7x - 18)$
$5x^6(x+ 9)(x - 2)$

25. $11x^4y + 33x^3y - 198x^2y$
$11x^2y(x^2 + 3x - 18)$
$11x^2y(x + 6)(x - 3)$

26. $(a+ 2)(x^2 + 9x + 14)$
$(a+ 2)(x+ 2)(x+ 7)$

27. $(m - 3)(x^2 - 5x + 6)$
$(m - 3)(x - 2)(x - 3)$

28. $(x+ 5)(x^2 + 7x - 18)$
$(x+ 5)(x+ 9)(x - 2)$