Math 95--Factoring Worksheet #4--page 1

This handout will approach factoring two different types of binomials: the difference of squares is one type and the sum or difference of cubes is the other type.

A. Difference of Squares

a. First, review the following:

$(x + 10)(x - 10)$	conjugates
$x^2 - 10x + 10x - 100$	distribute
$x^2 - 100$	difference of squares

When you multiply conjugates, you get the difference of squares.

b. When you factor the difference of squares, you'll always get conjugates.

$x^2 - 100$

$(x + 10)(x - 10)$

c. This is the pattern: $a^2 - b^2 = (a + b)(a - b)$

Look at a few more examples and see if you can follow the thinking.

d. $z^2 - 49$

$(z + 7)(z - 7)$

e. $m^2 - 196$

$(m + 14)(m - 14)$

you can use the square root button to find $\sqrt{196} = 14$

f. $4y^2 - 25$

$(2y + 5)(2y - 5)$

g. $36a^2 - 121z^2$

$(6a + 11z)(6a - 11z)$

h. $x^4 - 49y^2$

$(x^2 + 7y)(x^2 - 7y)$

i. $45y^2 - 20$ GCF first!

$5(9y^2 - 4)$

$5(3y + 2)(3y - 2)$

Factor the following.

1.	$x^2 - 1$	2.	$a^2 - 4$	3.	$m^2 - 9$	4.	$z^2 - 16$
5.	$y^2 - 25$	6.	$k^2 - 36$	7.	$b^2 - 49$	8.	$c^2 - 64$
9.	$w^2 - 81$	10.	$x^2 - 100$	11.	$4y^2 - 9$	12.	$16z^2 - 25$
13.	$81a^2 - 49$	14.	$36m^2 - 121$	15.	$25z^2 - 4$	16.	$2x^2 - 8$
17.	$3k^2 - 75$	18.	$5w^2 - 45$	19.	$4a^2 - 16$	20.	$4b^2 - 36$
21.	$25c^2 - 100$	22.	$x^2 - y^2$	23.	$x^2 - 25y^2$	24.	$a^2 - 36m^2$
25.	$49y^2 - 25z^2$	26.	$81a^2 - 4b^2$	27.	$18x^2 - 8y^2$	28.	$a^4 - 16b^2$

B. Sum or Difference of Cubes First, look at the following to find the pattern:

j.

$1^3 = 1$	so	$\sqrt[3]{1} = 1$	$4^3 = 64$ so $\sqrt[3]{64} = 4$
$2^3 = 8$	so	$\sqrt[3]{8} = 2$	$5^3 = 125$ so $\sqrt[3]{125} = 5$
$3^3 = 27$	so	$\sqrt[3]{27} = 3$	$6^3 = 216$ so $\sqrt[3]{216} = 6$

Math 95--Factoring Worksheet #4--page 2

The following are examples of the sum or difference of cubes:

$x^3 + 8$
$x^3 + 27$
$x^3 - 64$
$x^3 - 125$
$8x^3 + 125$
$27x^3 - 64$

Some books teach factoring the sum/difference of cubes by using a "formula". Observe.

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

You'll notice there are similarities between the two. Also, you always get a binomial times a trinomial.

I usually think about factoring the sum or difference of cubes in word form. I get the binomial first (cube root the first term, keep the sign, cube root the second term). Then I build the trinomial from the binomial (square the first term, multiply the terms together and change the sign, square the second term so it's always a positive). It's easier with an example.

k. $x^3 + 8$
$(x + 2)(x^2 - 2x + 4)$

Binomial:
cube root of $x^3 = x$
keep plus sign
cube root of $8 = 2$
$(x + 2)$

Trinomial:
square $x = x^2$
multiply x and 2 = 2x and change sign so $-2x$
square $2 = 4$ (notice it's positive 4)
$(x^2 - 2x + 4)$

Factors:
$(x + 2)(x^2 - 2x + 4)$

Check:
$(x + 2)(x^2 - 2x + 4)$
$x^3 - 2x^2 + 4x + 2x^2 - 4x + 8$
$x^3 + 8$

l. $x^3 - 125$
$(x - 5)(x^2 + 5x + 25)$

Binomial:
cube root of $x^3 = x$
keep minus sign
cube root of $125 = 5$
$(x - 5)$

Trinomial:
square $x = x^2$
multiply x and $-5 = -5x$ and change sign so $+5$
square $-5 = (-5)(-5) = 25$ (notice it's a positive 25)
$(x^2 + 5x + 25)$

Factors:
$(x - 5)(x^2 + 5x + 25)$

Check:
$(x - 5)(x^2 + 5x + 25)$
$x^3 + 5x^2 + 25x - 5x^2 - 25x - 125$
$x^3 - 125$

Math 95--Factoring Worksheet #4--page 3

m. $64x^3 + 27$

$(4x + 3)(16x^2 - 12x + 9)$

Binomial: cube root of $64x^3 = 4x$
keep plus sign
cube root of $27 = 3$
$(4x + 3)$

Trinomial: square $4x = 16x^2$
multiply $4x$ and $3 = 12x$ and change sign so $-12x$
square $3 = 9$
$(16x^2 - 12x + 9)$

Factors: $(4x + 3)(16x^2 - 12x + 9)$

Check: $(4x + 3)(16x^2 - 12x + 9)$
$64x^3 - 48x^2 + 36x + 48x^2 - 36x + 27$
$64x^3 + 27$

n. $64x^3 + 27$ Let's re-do this problem using the formula.

$(4x)^3 + (3)^3$ First write the bases to the 3rd power

$(4x + 3)[(4x)^2 - (4x)(3) + (3)^2]$ Use the formula $(a + b)(a^2 - ab + b^2)$

$(4x + 3)(16x^2 - 12x + 9)$ Simplify, and notice you get the same answer as you did in example m!

You try these.

29.	$x^3 + 1$	30.	$x^3 - 8$	31.	$x^3 + 27$	32.	$x^3 - 64$
33.	$x^3 + 125$	34.	$8x^3 - 27$	35.	$27x^3 + 64$	36.	$125x^3 - 27$

Answer Key.

1.	$(x+1)(x-1)$	2.	$(a+2)(a-2)$	3.	$(m+3)(m-3)$
4.	$(z+4)(z-4)$	5.	$(y+5)(y-5)$	6.	$(k+6)(k-6)$
7.	$(b+7)(b-7)$	8.	$(c+8)(c-8)$	9.	$(w+9)(w-9)$
10.	$(x+10)(x-10)$	11.	$(2y+3)(2y-3)$	12.	$(4z+5)(4z-5)$
13.	$(9a+7)(9a-7)$	14.	$(6m+11)(6m-11)$	15.	$(5z+2)(5z-2)$
16.	$2(x+2)(x-2)$	17.	$3(k+5)(k-5)$	18.	$5(w+3)(w-3)$
19.	$4(a+2)(a-2)$	20.	$4(b+3)(b-3)$	21.	$25(c+2)(c-2)$
22.	$(x+y)(x-y)$	23.	$(x+5y)(x-5y)$	24.	$(a+6m)(a-6m)$
25.	$(7y+5z)(7y-5z)$	26.	$(9a+2b)(9a-2b)$	27.	$2(3x+2y)(3x-2y)$
28.	$(a^2+4b)(a^2-4b)$	29.	$(x+1)(x^2-x+1)$	30.	$(x-2)(x^2+2x+4)$
31.	$(x+3)(x^2-3x+9)$	32.	$(x-4)(x^2+4x+16)$	33.	$(x+5)(x^2-5x+25)$
34.	$(2x-3)(4x^2+6x+9)$	35.	$(3x+4)(9x^2-12x+16)$	36.	$(5x-3)(25x^2+15x+9)$